Predictive Power-Balance Modeling of PEGASUS and NSTX-U Local Helicity Injection Discharges

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PEGASUS Toroidal Experiment

Abstract

Local helicity injection (LHI) with outer poloidal-field (PF) induction for solenoid-free startup is being studied on Pegasus, reaching $I_p \leq 0.175$ MA with 6 kA of injected current. A lumped-parameter circuit model for predicting the performance of LHI initiated plasmas is under development. The model employs energy and helicity balance, and includes applied PF ramping and the inductive effects of shape evolution. Low-A formulations for both the plasma external inductance and a uniform equilibrium-field are used to estimate inductive voltages. Pegasus LHI plasmas are created near the outboard injectors with aspect ratio (A) ≈ 5-6.5 and grow inward to fill the confinement region at A ≤ 1.3. Initial results match these experimental $I_p(t)$ trajectories within 15 kA with a prescribed geometry evolution. Helicity injection is the largest driving term in the initial phase, but in the later phase is reduced to 20-45% of the total drive as PF induction and decreasing plasma inductance become dominant. In contrast, attaining ~1 MA non-solenoidal startup via LHI on NSTX-U will require operation in the regime where helicity injection drive exceeds inductive and geometric changes at full size. A large-area multi-injector array will increase available helicity injection by 3-4 times and allow exploration of this helicity-dominated regime at I_p ~0.3 MA in Pegasus. Comparison of model predictions with time-evolving magnetic equilibria is in progress for model validation.

*Work supported by US DOE Grant DE-FG02-96ER54375

Outline

- Pegasus uses localized, electron-current sources for Local Helicity Injection Current Drive (LHI-CD)
  - Maximum achievable plasma currents with LHI predicted by Taylor Relaxation principles, helicity balance

- A predictive, lumped-parameter, power-balance model is under development for Pegasus LHI discharges
  - A predictive model is desired for injector and scenario development
  - LHI discharge early evolution merits further experimental study for better predictive capability

- Model predicts large-area injectors necessary to reach performance goals scalable to ~1 MA startup on NSTX-U

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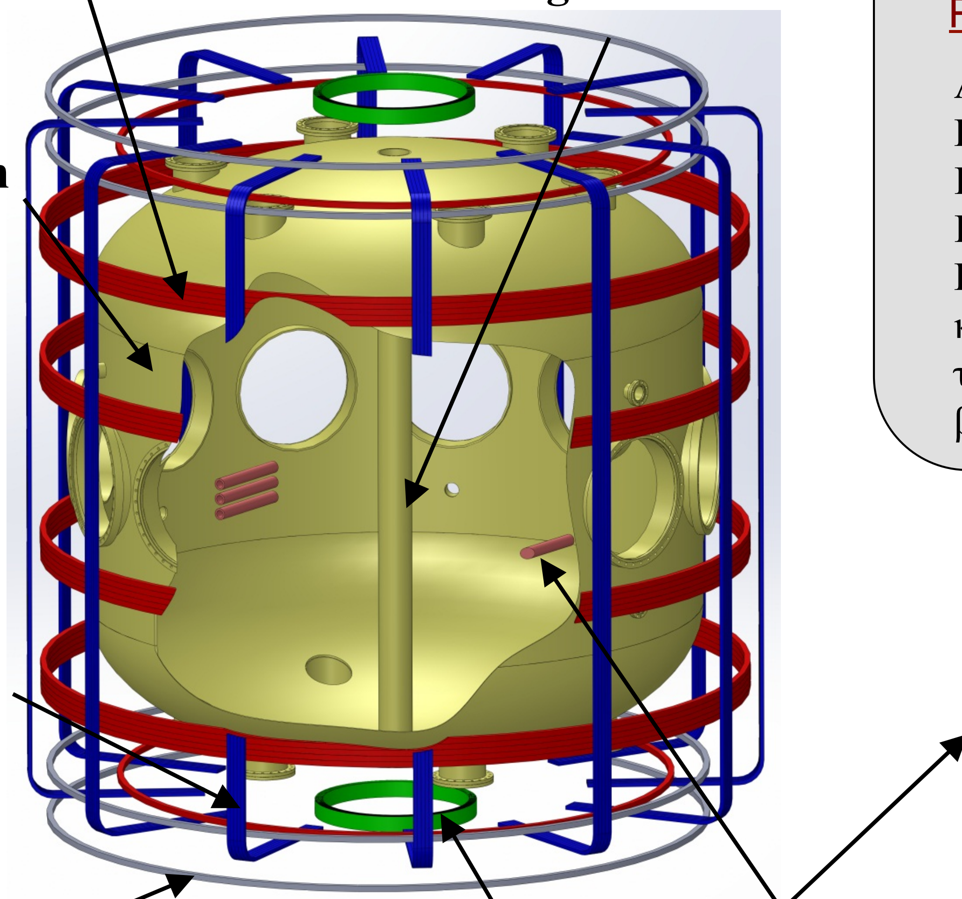
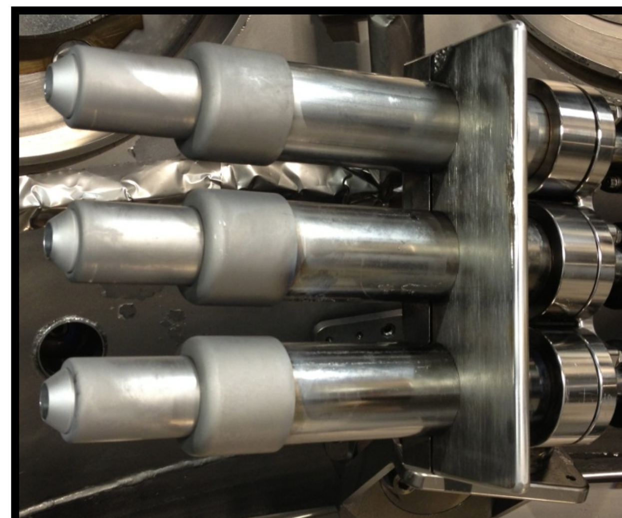
PEGASUS is a Compact, Ultralow-A ST



Experimental Parameters

Parameter	Achieved	Goals
A	1.15 – 1.3	1.12 – 1.3
R(m)	0.2 – 0.45	0.2 – 0.45
I_p (MA)	$\leq$ .23	$\leq$ 0.30
I_N (MA/m-T)	6 – 14	6 – 20
RB_t (T-m)	$\leq$ 0.06	$\leq$ 0.1
κ	1.4 – 3.7	1.4 – 3.7
τ_{shot} (s)	$\leq$ 0.025	$\leq$ 0.05
β_t (%)	$\leq$ 25	> 40



Helicity Injection is Tokamak Current Drive

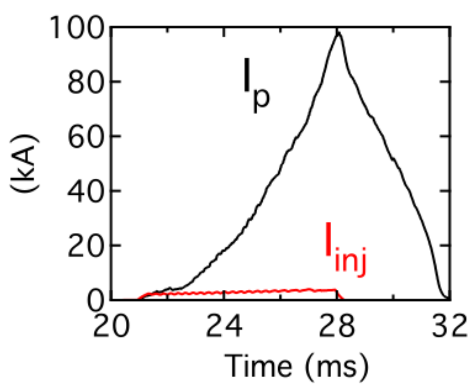
Total helicity K in a Tokamak geometry: $K = \int_V (\mathbf{A} + \mathbf{A}_{vac}) \cdot (\mathbf{B} - \mathbf{B}_{vac})\, d^3x$

$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J} \cdot \mathbf{B}\, d^3x - 2\frac{\partial \psi}{\partial t}\Psi - 2\int_A \Phi \mathbf{B} \cdot d\mathbf{s}$$

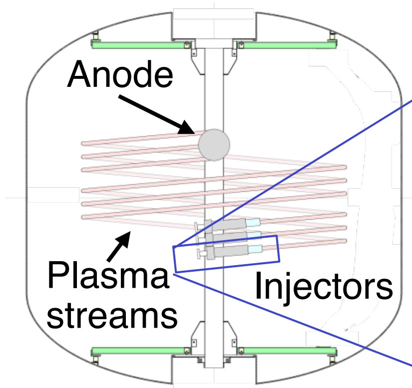
- Resistive Helicity Dissipation
  - $\mathbf{E} = \eta \mathbf{J} \rightarrow$ much slower than energy dissipation ($\eta \mathbf{J}^2$)
  - Turbulent relaxation processes dissipate energy and conserve helicity
- AC Helicity Injection: $\dot{K}_{AC} = -2\frac{\partial \psi}{\partial t}\Psi = 2V_{loop}\Psi$
  - Ψ is toroidal flux, ψ is poloidal flux (*e.g.*, current drive through solenoid induction)
- DC Helicity Injection $\dot{K}_{DC} = -2\int_A \Phi \mathbf{B} \cdot d\mathbf{s} = 2V_{bias} B_{\perp} A_{inj}$
  - Φ is electrostatic potential

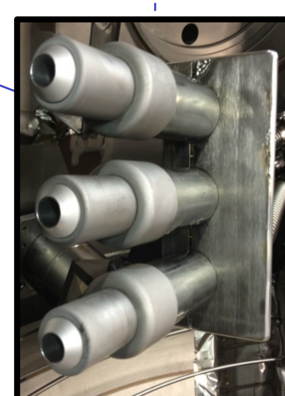
PEGASUS Uses DC Helicity Injection with Edge Localized Current Injectors

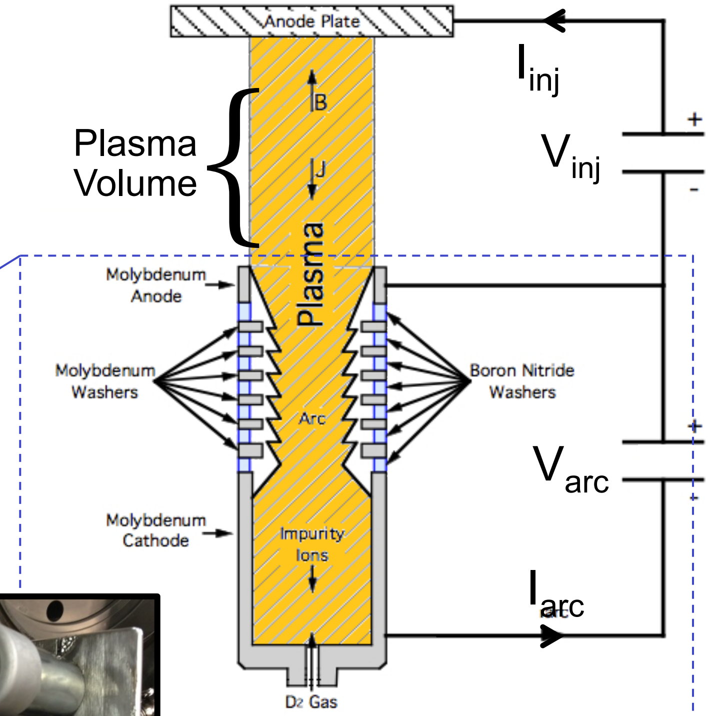
- Goal: Scalable, non-solenoidal startup technique
- Assisted by PF-ramping for additional current drive



Current multiplication many times injected current*

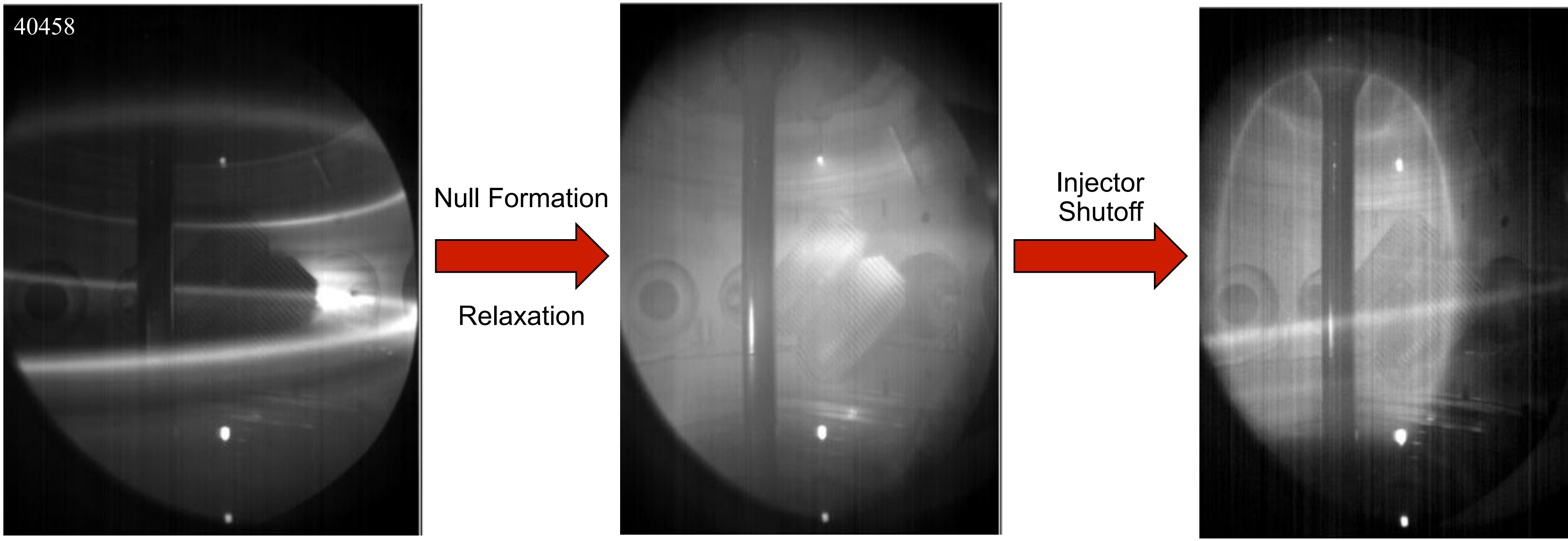






*A. J. Redd, et al., (2009) Journal of Fusion Energy, **28**

LHI Injects Current Streams that Relax, Form Tokamak-Like Plasma

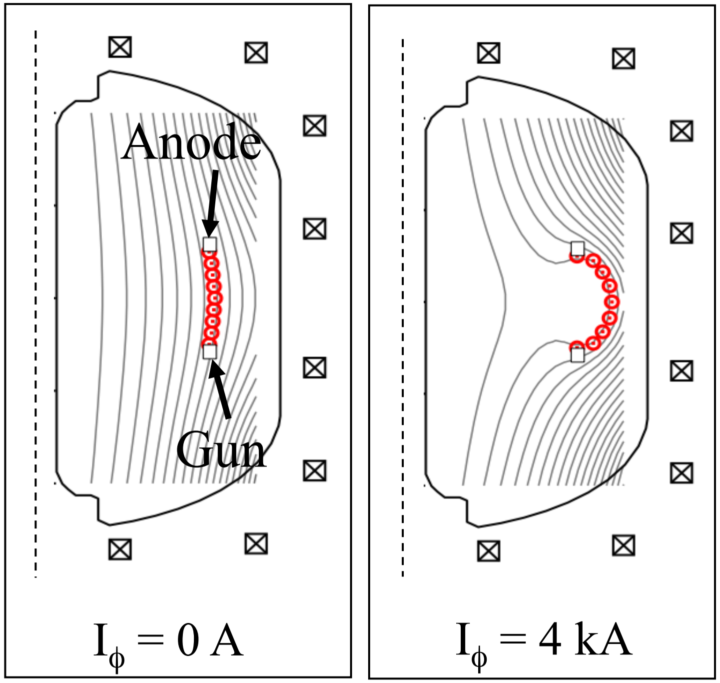


- Current injected along helical vacuum field
  - Local, active current sources
- MHD relaxation, tokamak-like state
  - Onset via local PF null
  - Constrained by helicity, Taylor relaxation limits
- Tokamak plasmas produced after injector shut off
  - Couples to alternative current drive sources

Battaglia et al., Nucl. Fusion ***51****, 073029 (2011)*

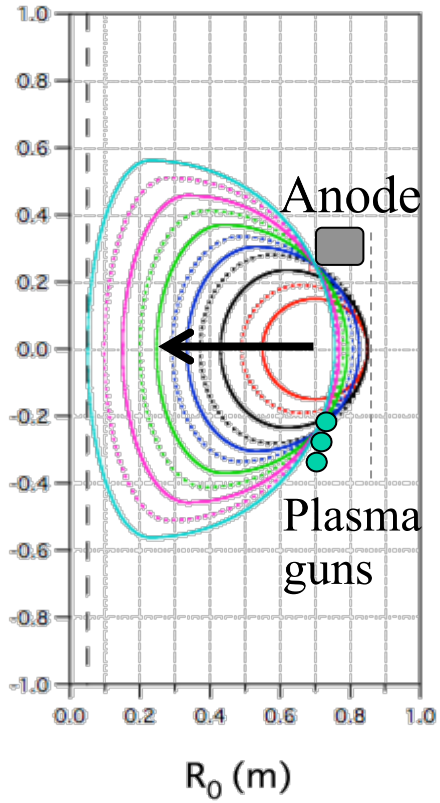
Pegasus LHI Plasmas Expand from Outboard Injectors to Low-A, Highly-Shaped

- Initial relaxation to Tokamak-like state coincident with field-null
  - Explained experimental minimum I_{inj}/B_v
- Plasma expands inward to fill confinement region



2-D force free current model

**D. J. Battaglia, et al., Nucl. Fusion, 51, 073029, 2011*



Helicity Balance, Taylor Relaxation Criteria Set Maximum Achievable I_p from LHI

Helicity balance in a tokamak geometry:

$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J}\cdot\mathbf{B}\, d^3x - 2\frac{\partial\psi}{\partial t}\Psi - 2\int_A \Phi\mathbf{B}\cdot d\mathbf{s} \quad \Rightarrow \quad I_p \le \frac{A_p}{2\pi R_0\langle\eta\rangle}\left(V_{ind} + V_{eff}\right)$$

- Helicity injection can be expressed as an effective loop voltage
- I_p limit depends on the plasma confinement via resistivity η

$$V_{eff} \approx \frac{A_{inj}B_{\phi,inj}}{\Psi_T}V_{inj}$$

Taylor relaxation of a force-free equilibrium:

$$\nabla\times B = \mu_0 J = \lambda B$$

$$\lambda_p \le \lambda_{edge}$$

$$\Rightarrow \quad \frac{\mu_0 I_p}{\Psi_T} \le \frac{\mu_0 I_{inj}}{2\pi R_{inj} w B_{\theta,inj}} \quad \Rightarrow \quad I_p \le \left[\frac{C_p}{2\pi R_{inj}\mu_0}\frac{\Psi_T I_{inj}}{w}\right]^{1/2}$$

Assumptions:

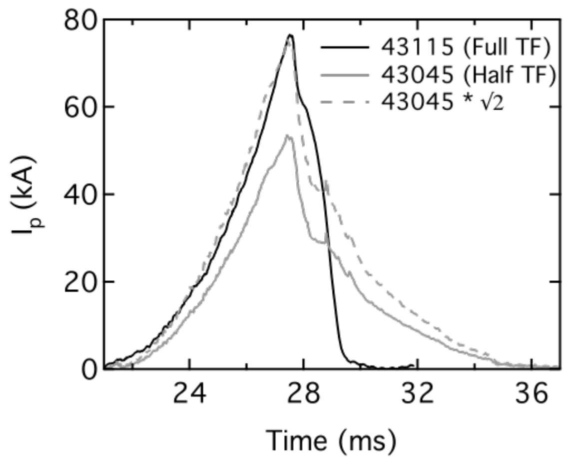
- Driven edge current mixes uniformly in SOL
- Fields average to Tokamak-like structure

A_p	Plasma area
C_p	Plasma circumference
Ψ_T	Plasma toroidal flux
w	Edge current channel width

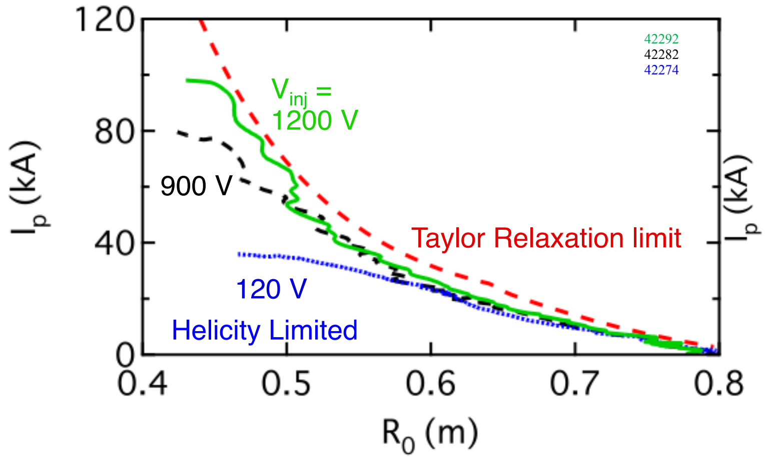
Experimental $I_{p,max}$ Follows Taylor Limit Scalings

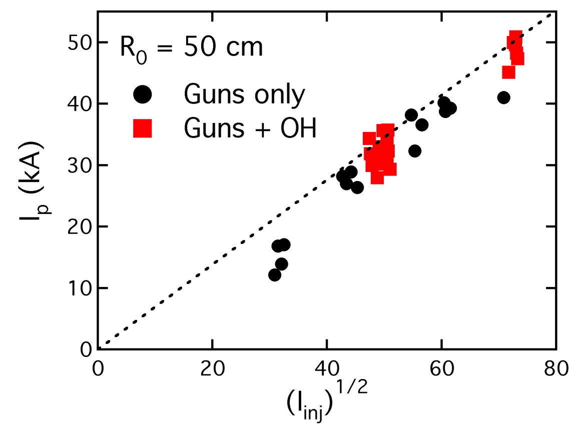
- Expected Taylor Limit scalings with I_{inj} and I_{TF} confirmed
  - A hard limit as long as in MHD turbulent state

$$I_{p,max} \propto \sqrt{I_{TF} I_{inj}}$$



- $I_p(R,t)$ trajectory approaches Taylor relaxation limit with sufficient LHI, PF-induction drive





Battaglia *et al*., Nucl. Fusion **51**, 073029 (2011)

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A Predictive Model for LHI Startup is Desired

- The Taylor relaxation current limit gives $I_{p,max}$, but does not prescribe the required V_{eff} to meet it
- Estimates of required HI rate are needed to design injector systems capable of meeting performance goals
- A power-balance model is in development to guide injector and scenario development
  - Verification, comparison to time-evolving reconstructions and additional measurements is underway

System Differential Equation is Solved Numerically with Runge-Kutta Algorithm

- The system simplifies to the differential equation:

$$\frac{\partial I_p}{\partial t} = \frac{-\frac{1}{2}\frac{\partial L_p}{\partial t} - R_p + \frac{\partial}{\partial t}\left[\pi R_0^2\left(\frac{B_v}{I_p}\right)\right]}{L_p - \pi R_0^2\left(\frac{B_v}{I_p}\right)} I_p + \frac{V_{OH} + V_{eff}}{L_p - \pi R_0^2\left(\frac{B_v}{I_p}\right)}$$

$$\text{where} \quad L_P = L_e + L_i$$

- This initial value problem is solved numerically via the Runge-Kutta method using the Igor Pro v6.32A software

Low-A, Analytic Force-Balance B_v Formula Used to Approximate PF Loop-Voltage

- Applied vertical field provides force-balance and inductive loop-voltage
  - Loop-voltage approximated by uniform vertical-field for force-balance at R_0:

$$V_{PF} = \frac{\partial}{\partial t}\left(\sum_i \psi_{PF,i}\right) \approx \frac{\partial}{\partial t}\left[\pi R_0^2 \, B_V\big|_{R_0}\right]$$

- B_v required for force-balance is aspect-ratio and shape dependent
  - Discharges evolve from high-A to low-A, and circular to strongly shaped

- Uses Mitarai & Takase* formula for B_v for force-balance at low-A with **κ**:

$$B_V = \frac{\mu_0 I_p}{4\pi R_0}\left\{\frac{1}{\mu_0}\frac{\partial L_e}{\partial R} + \frac{\ell_i}{2} + \beta_p - \frac{1}{2}\right\}$$

*O. Mitarai and Y. Takase 2003 *Fusion Sci. Technol.*

Resistivity is Explicitly Input in the Model

- For now, Spitzer resistivity* assumed:

$$R_p = \eta \frac{2\pi R_0}{A_p} \qquad \eta = 1.65 \times 10^{-9} \frac{Z_{eff}}{T_{e[keV]}^{3/2}} \ln \Lambda$$

*L. Spitzer 1962 *The physics of fully-ionized gases* (2nd Edn.) Interscience, NY

*J. Wesson 2004 *Tokamaks*

- Improved accuracy awaits improved diagnostics
  - $Z_{eff} \sim 1$ for Pegasus LHI discharges
  - Neoclassical effects currently not included
  - Thomson Scattering diagnostic under development:
    - See poster:

      D.J. Schlossberg "Commissioning of Thomson Scattering on the Pegasus Toroidal Experiment" TP8.00023

Poynting's Theorem Applied to Model Tokamak Power-Balance

$$I_p V_s = \frac{\partial}{\partial t}\left(W_p\right) + I_p^2 R_p - I_p V_{NICD}$$

Plasma surface voltage

- Force-balance explicitly enforced
- Inductive current-drive from OH, PF-ramping
- Plasma external inductance

$$V_s = \frac{\partial}{\partial t}\left(\psi_{OH} + \sum_i \psi_{PF,i}\right) - \frac{1}{I_p}\frac{\partial}{\partial t}\left(\frac{1}{2}L_e I_p^2\right)$$

Internal magnetic energy

$$\frac{1}{I_p}\frac{\partial}{\partial t}\left(W_p\right) = \frac{1}{I_p}\frac{\partial}{\partial t}\left(\frac{1}{2}L_i I_p^2\right)$$

Resistive Dissipation

$$V_R = I_p R_p = I_p\left(\frac{\eta_p 2\pi R_0}{A_p}\right)$$

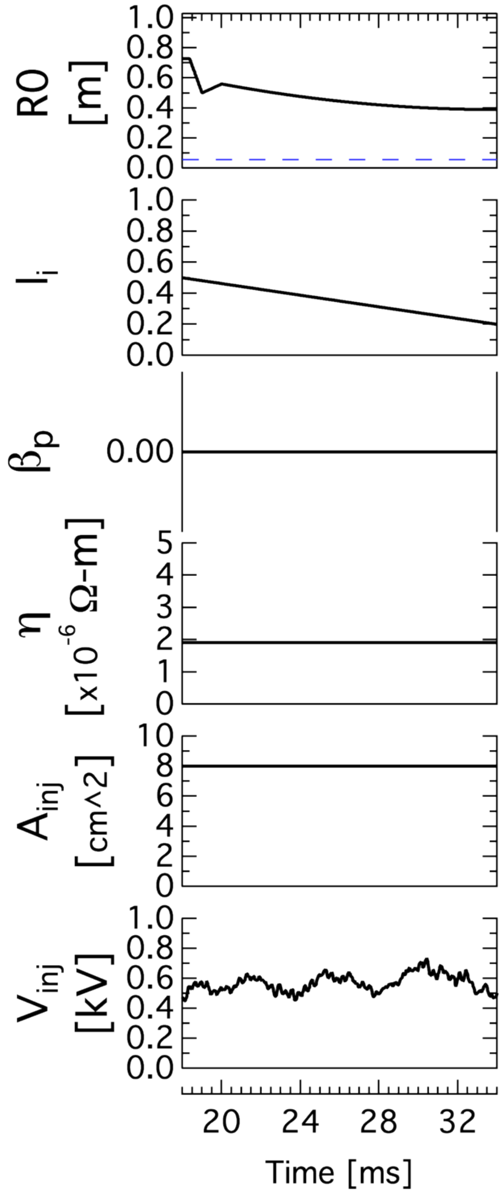
Non-inductive current drive

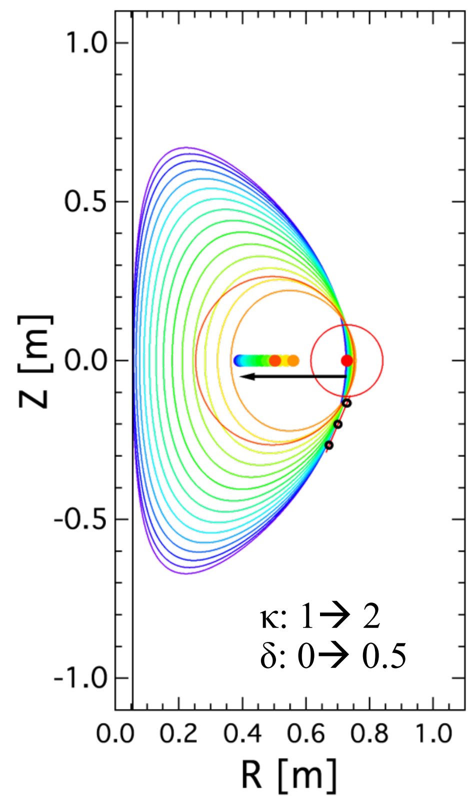
$$V_{NICD} = V_{eff}$$

*J.A. Romero and JET-EFDA Contributors 2010 *Nucl. Fusion* **50** 115002
*S.P. Hirshman and G.H. Nielson 1986 *Phys. Fluids* **29** 790
*S. Ejima et al 1982 *Nucl. Fusion* **22** 1313

Lumped-Parameter Inputs Constitute Plasma Parameters

- $I_p(t_0)$
  - Provides initial condition to DE solver
- Shape
  - $R_0(t)$, $a(t)$, $\kappa(t)$, $\delta(t)$
  - Vertical symmetry ($Z_0(t)=0$)
- $\eta(t)$, $\ell_i(t)$, $\beta_p(t)$
  - Assumed constant in time for now:
    $\eta=\eta_{Spitzer}(Z_{eff}=1, T_e=60eV, \ln\Lambda=17)$
    $\beta_p=0$
  - ℓ_i typically dropping: 0.5 → 0.2
- $V_{eff}(t) \sim A_{inj}(t)*V_{inj}(t)$
  - A_{inj} of helicity injectors





Plasma Inductance Includes Analytic, Low-A External Inductance Approximation

- Plasma self-inductance can be described as a combination of internal and external inductances

$$L_p = L_e + L_i$$

- Plasma external inductance is heavily aspect-ratio dependent

$$L_e = \mu_0 R_0 \frac{a(\varepsilon)(1-\varepsilon)}{1-\varepsilon+\kappa b(\varepsilon)}$$

$$a(\varepsilon) = \left(1+1.81\sqrt{\varepsilon}+2.05\varepsilon\right)\ln\left(\frac{8}{\varepsilon}\right)-\left(2.0+9.25\sqrt{\varepsilon}+1.21\varepsilon\right)$$

$$b(\varepsilon) = 0.73\sqrt{\varepsilon}\left(1+2\varepsilon^4-6\varepsilon^5+3.7\varepsilon^6\right)$$

*S.P. Hirshman and G.H. Nielson 1986 *Phys. Fluids* **29** 790

- Internal inductance typically assumed decreasing during the discharge (typically ℓ_i = 0.5→0.2)

$$W_p = \iiint_{V_p} \frac{B_\theta^2}{2\mu_0} dV = \frac{1}{2} L_i I_p^2$$

$$\ell_i \equiv \frac{2L_i}{\mu_0 R_0}$$

LHI Provides Non-Inductive Current Drive

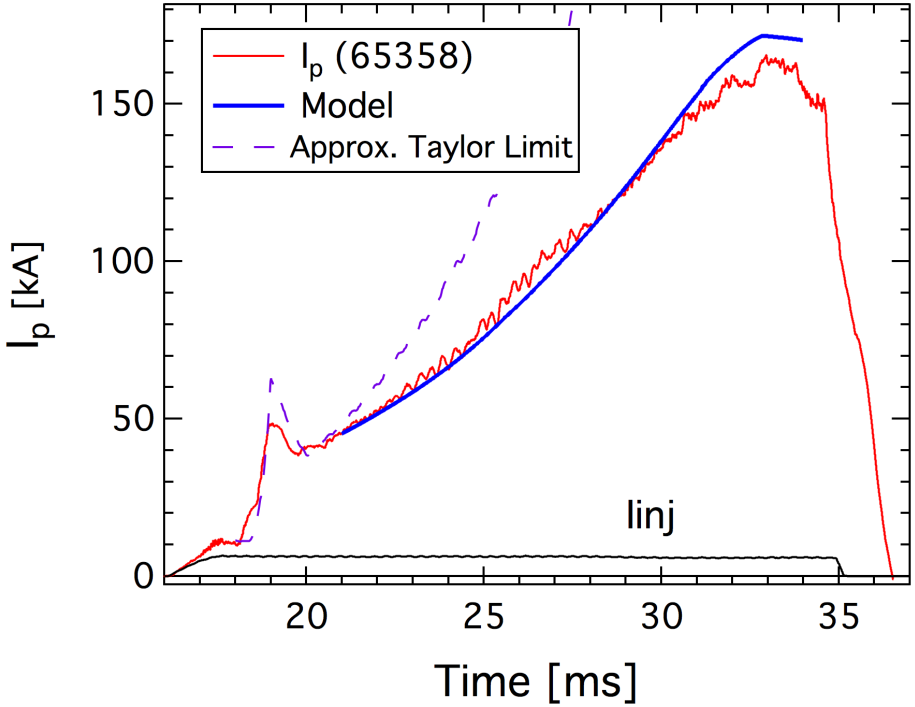
- LHI current drive incorporated through V_{NICD} with V_{eff}:

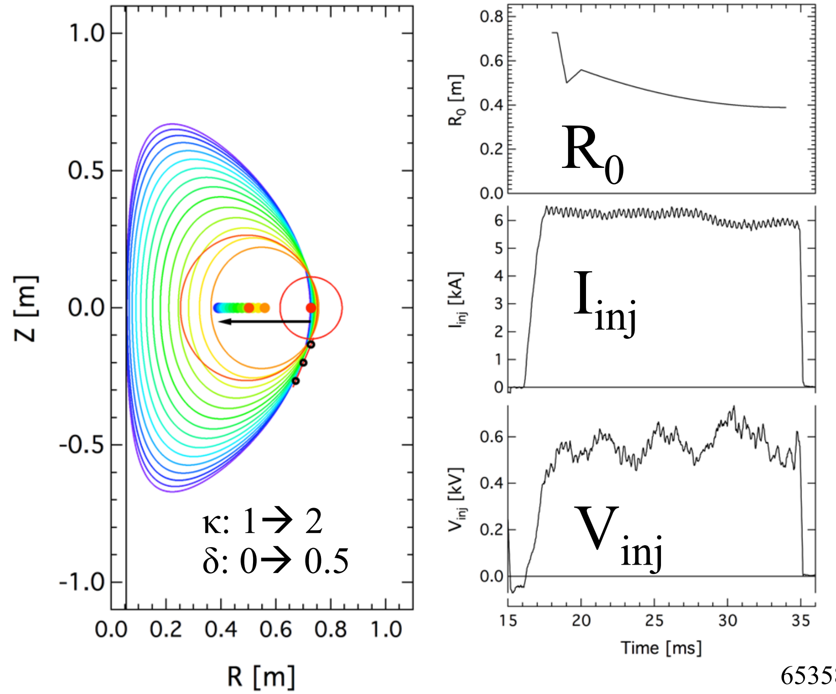
$$V_{NICD} = V_{eff} \approx \frac{A_{inj} B_{\phi,inj}}{\Psi_T} V_{bias}$$

- Effectiveness of LHI is dependent on plasma, injector geometry

- This model provides a tool for designing injectors and power-supplies

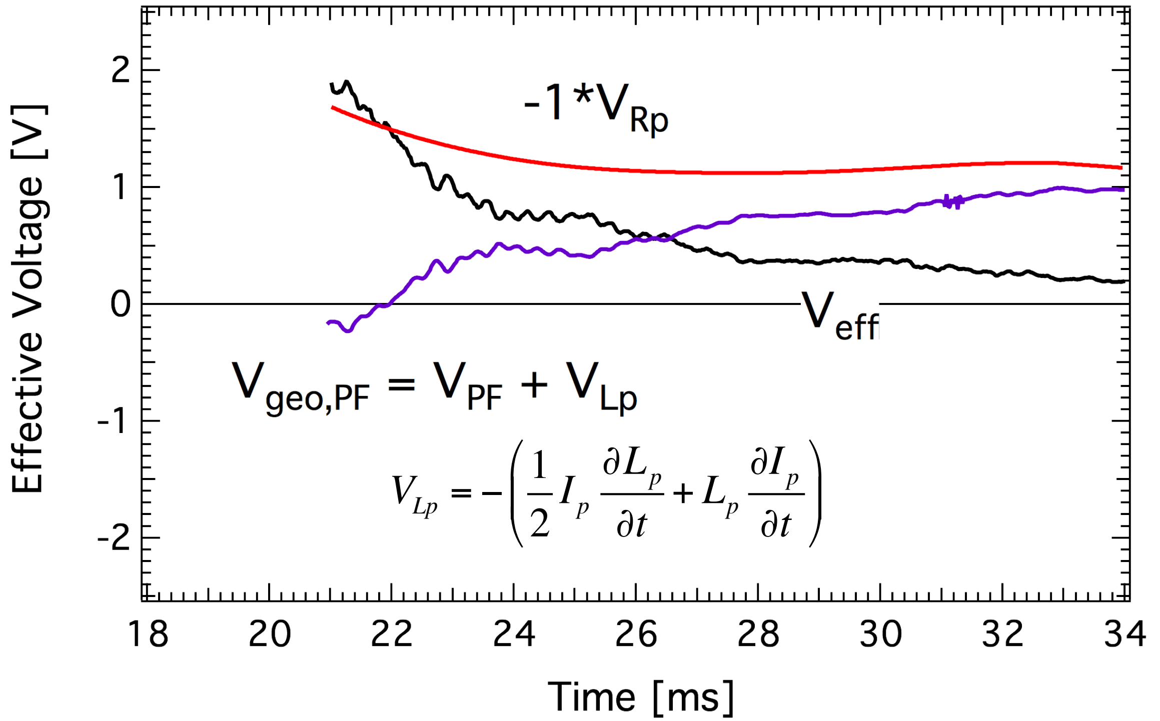
Power-Balance Analysis Gives Reasonable Reproductions of Pegasus LHI Discharges





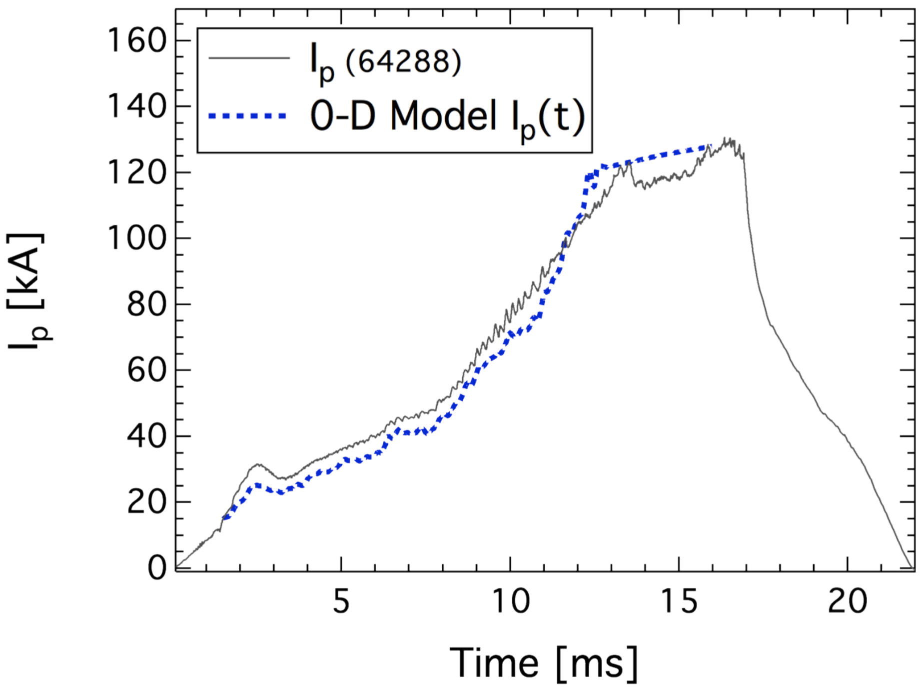
- Analysis reasonably follows I_p trajectory of modeled discharges
  - Early stages of plasma evolution likely limited by Taylor Relaxation current limit
- Plasma R_0 estimated from 2 external magnetic coils
  - Adjusted based on imaging if necessary
  - Time-evolving equilibria underway

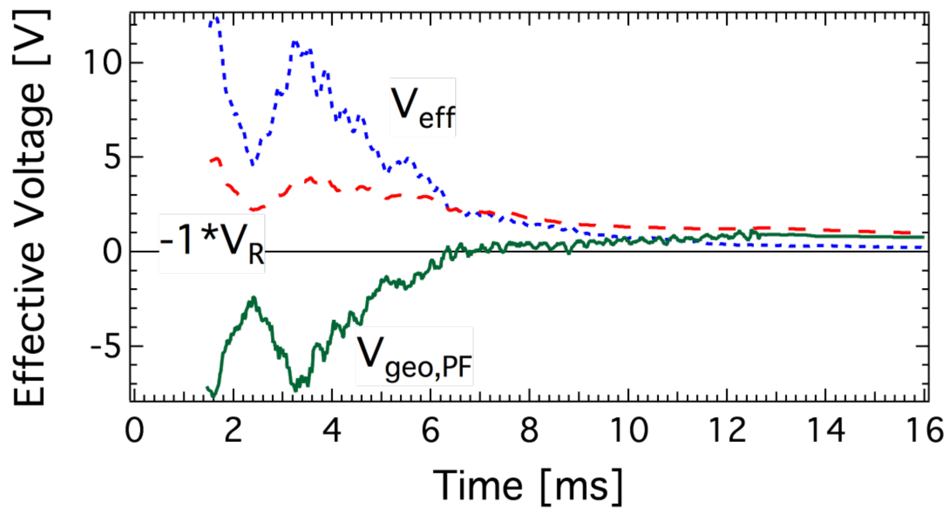
Pegasus LHI Discharges are Dominated by Inductive Effects As the Plasma Grows

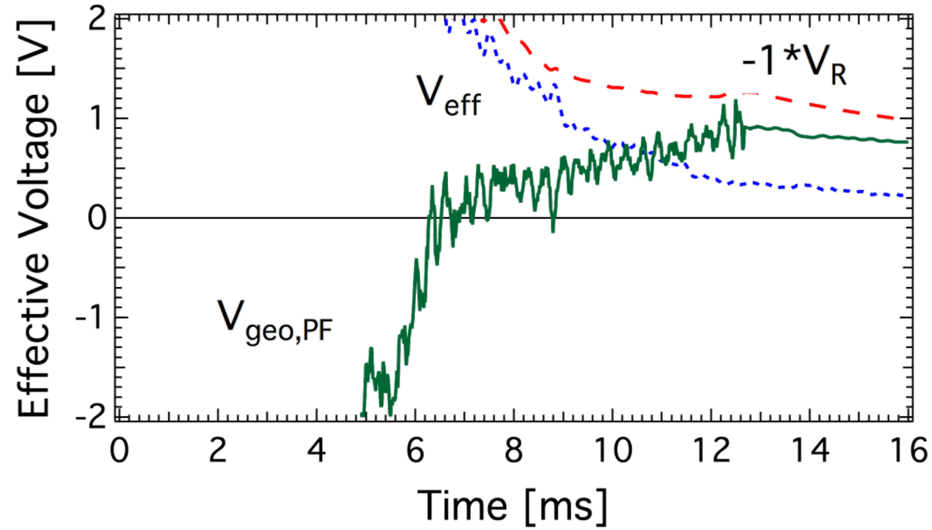


- LHI provides significant current drive to early, outboard-limited plasma
- PF induction, dropping ℓ_i and increasingly necessary as plasma grows to inboard in Pegasus

Power-Balance Analysis Gives Reasonable Reproductions of Pegasus LHI Discharges



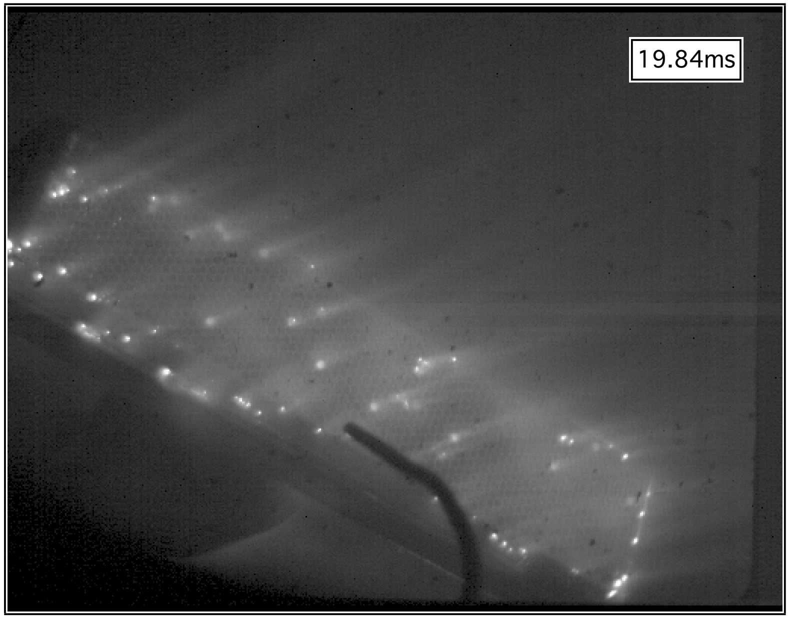




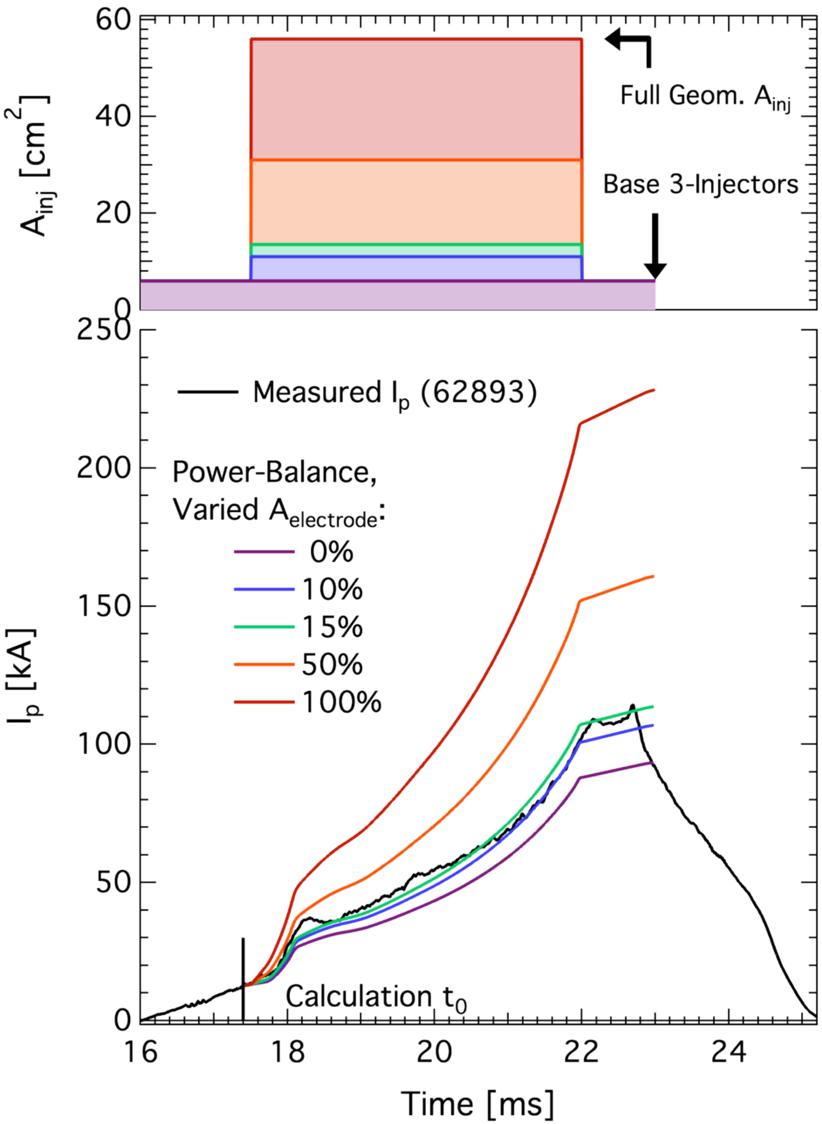
- Analysis reasonably follows I_p trajectory of modeled discharges

LHI Power-Balance Model Applied to Evaluate Gas-Effused Electrode Performance

- Large-A_{inj} electrode intended to provide high V_{eff}
  - ~ 50 cm^2 Mo plate with uniform gas flow provided through ~ 2mm holes
  - Relies on hollow cathode formation from immersion in plasma SOL
- Narrow hollow-cathode operating space limited performance
  - Presence of cathode-spot activity suggested reduction in A_{inj}



Reduced A_{inj} in Passive Electrode Consistent with Reduced V_{eff}, Plasma Performance



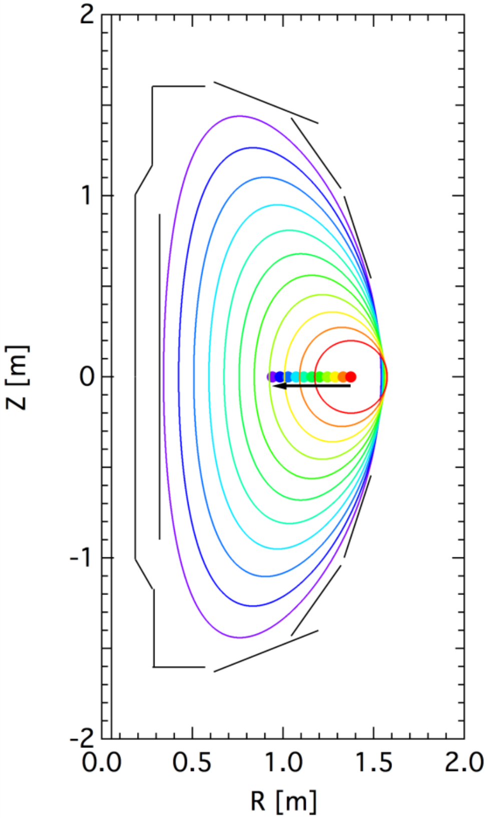
- Injector area varied in model to quantify approximate area utilization:
  - Result: 10-15% consistent with I_p evolution
  - Qualitatively consistent with fractional electrode illumination
- Conclusion: passive electrode not useful as robust tool for high V_{eff}
  - Implies local arc-plasma sources useful for sustained V_{eff}

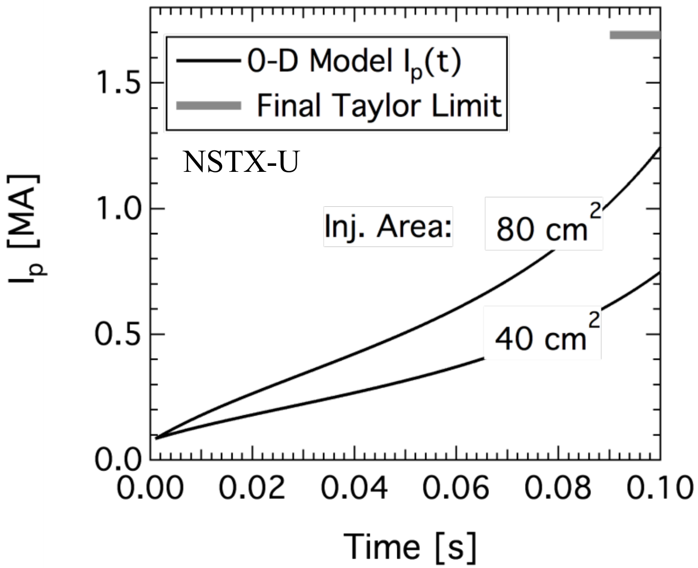
See Poster: J.M. Perry, *et al* "Local Helicity Injection Systems for Non-Solenoidal Startup in the Pegasus Toroidal Experiment" TP8.00019

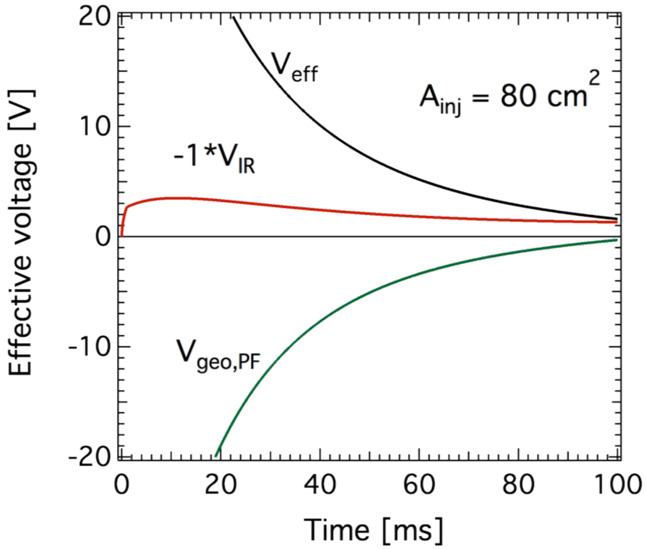
High Performance LHI Startup Requires Large-A_{inj} Helicity Injectors

- Goal: Scalable, non-solenoidal startup technique
- Initial modeling of NSTX-U suggests LHI drive must dominate discharge evolution to achieve ~1 MA startup
- In typical Pegasus LHI operations, PF-ramping and changing plasma inductance is a large source of current drive
- Pegasus operations at I_p~300 kA also require LHI current drive dominance
  - High current LHI operation on Pegasus should provide a test of the LHI-CD dominated regime
  - Would provide experimental verification of available V_{eff} drive from LHI, predictability of effective and required A_{inj} in injector sources

Model Applied to NSTX-U Geometry for Initial I_p~1 MA Start-up Scenario Prediction







- Significant LHI drive required to achieve I_p ~ 1 MA
  - Predicted $A_{inj}*V_{inj}$ requirement: ~ 80 cm^2 * 1kV

R_0: 1.37→0.94 κ: 1→ 2.4
A: 6.9→1.56 δ: 0→0.3
T_e: 150 eV

*C. Neumeyer *et all* (2009) *23rd IEEE/NPSS Symposium on Fusion Engineering*

**C. Neumeyer (2001) "NSTX Internal Hardware Dimensions"
http://nstx.pppl.gov/nstx/Engineering/NSTX_Eng_Site/Technical/Machine/NSTX_Engr_Machine_Dims_cm.html

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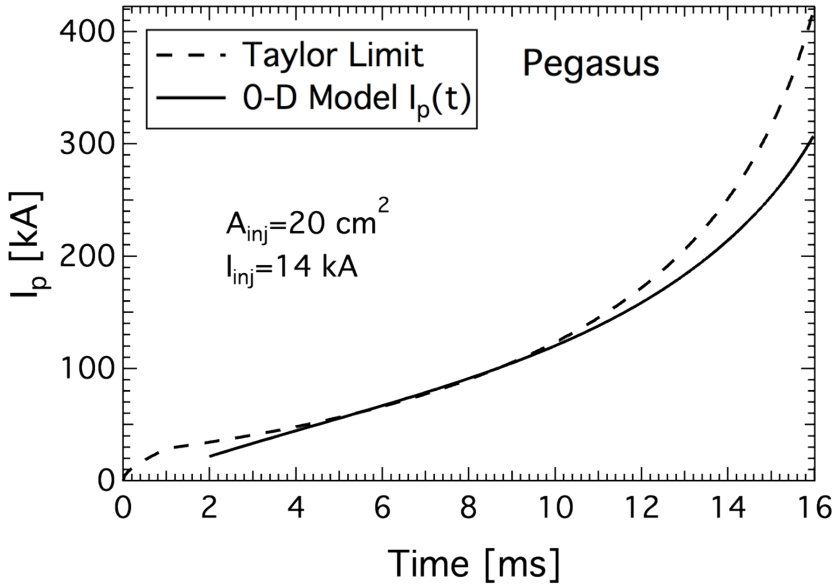
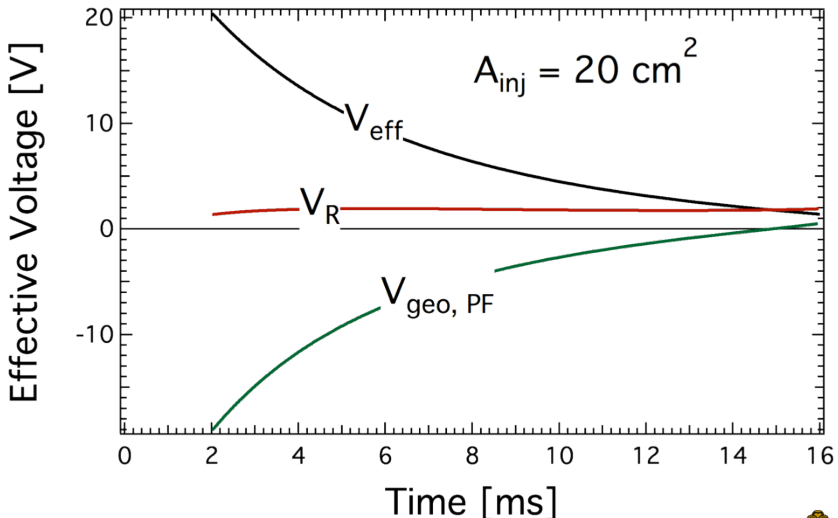
$I_p \geq 300$ kA on Pegasus in LHI-Drive Dominated Regime

- Modeling indicates Pegasus LHI discharges operating at $I_p \sim 300$ kA enter LHI-CD dominant regime
  - Expected at $A_{inj} \geq 20$ cm^2 with $V_{inj} = 1$kV
  - $I_{inj} = 14$ kA for sufficiently high Taylor Relaxation current limit

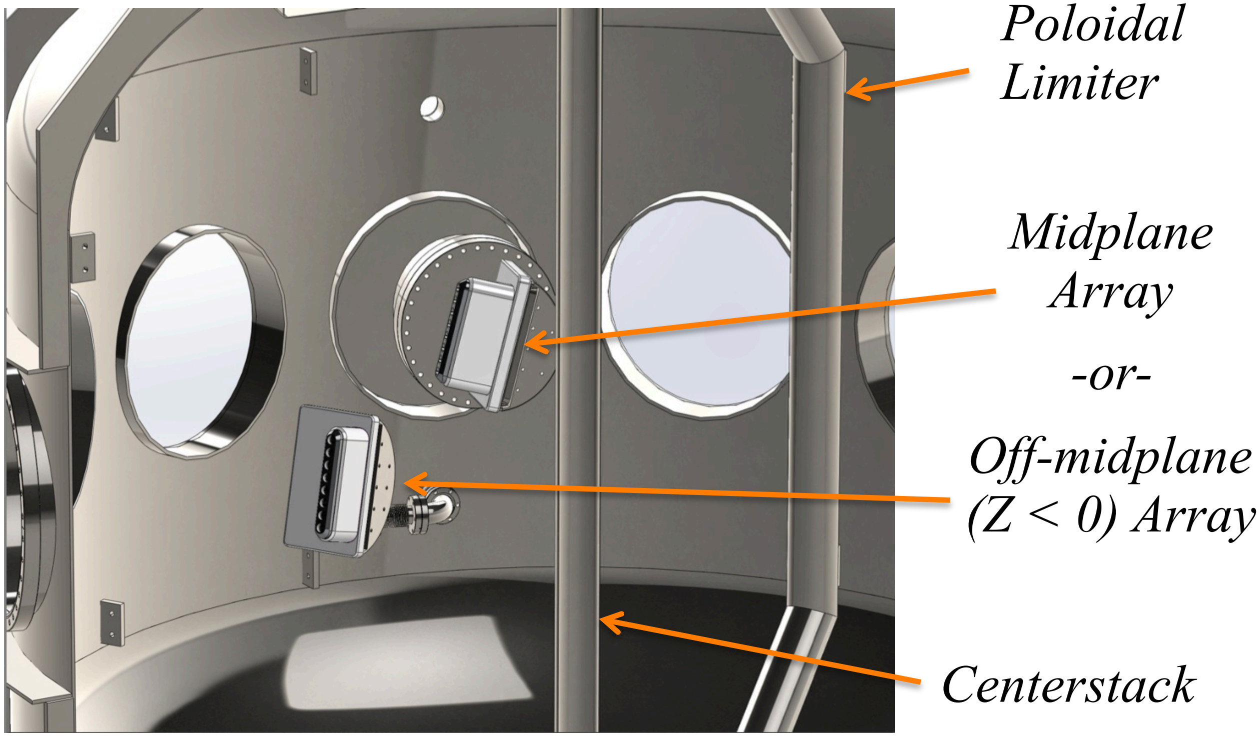


- High current operations will permit testing the regime where LHI drive dominates PF inductive drive



- Additional issues arise for larger machines
  - Injector heat loads at long-pulse
  - Possible injector aperture dependence on B_t

New 8-Injector Array in Fabrication to Test LHI-CD Dominated Regime



- 8-injector array will augment existing helicity injection capability
  - Provides an 16 cm^2 A_{inj}; combined $A_{inj} = 20cm^2$

See Poster: J.M. Perry, *et al* "Local Helicity Injection Systems for Non-Solenoidal Startup in the Pegasus Toroidal Experiment" TP8.00019

Outstanding Issues

- Test validity of constituent low-A approximations and plasma evolution assumptions
  - Ex: L_e, B_v
  - Testing against time-evolving equilibrium reconstructions, additional measurements is underway
- Investigation of Pegasus LHI plasma resistivity
  - Thomson Scattering diagnostic under development
- Experimental tests of V_{eff} predictions
  - New 8-injector array will provide substantial V_{eff} increase
- Appropriate choice of t_0
  - Taylor Relaxation and Kruskal-Shafranov limits suspected in early discharge evolution

Conclusions

- A predictive power-balance model for LHI startup being developed to guide injector, scenario development
  - Incorporates V_{eff} from helicity balance to include LHI current drive
  - More extensive validation effort planned
- Initial results reproduce Pegasus LHI-startup current trajectories reasonably well
  - Typical operation relies on substantial PF-induction for current drive
- Preliminary predictions suggest significant LHI drive to meet performance goals for startup on NSTX-U, Pegasus
  - Indicates operation in a regime where LHI drive dominates PF-induction
  - This regime is testable on the Pegasus ST