Power Balance Modeling of Local Helicity Injection for Non-Solenoidal ST Startup

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Layout Slide (Include for Posters)

12:1 scale Panel size: 8’ x 4’

US Legal 8.5 x 14”

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Populate with section titles, final slide titles

- Power Balance Modeling of Local Helicity Injection for Non-Solenoidal ST Startup
- Developing Predictive Capability for LHI Startup
- LHI is a promising Non-Solenoidal Startup Technique
- Local Plasma Sources Inject Current Streams that Reconnect to Form Tokamak-like Plasma
- Injector Location in LHI Emphasizes Different CD Mechanisms

- A Power-Balance Model for LHI Startup
- Ip Evolution predicted from Power-Balance and Taylor Relaxation
- Global Helicity Balance, Taylor Relaxation Limit Maximum I_p
- 0-D Power Balance Model Incorporates Analytic Plasma Inductance (ψ_{L_e}, ψ_{PF}) Formulae
- Analytic Plasma Inductance Formulas Expanded, Re-Calibrated for Non-uniform B_V, Ultralow-A
- Equilibrium-Calibrated Inductance Model Improves Estimates of Non-Solenoidal V_{IND}

- Comparison Between Different LHI Systems in Pegasus
- 0-D Model Takes Plasma, Injector Parameters as Inputs
- LFS Exhibit Taylor Limit Phase, HFS V_{LHI} Limited Through Entire Discharge
- LFS Discharges Driven Primarily by V_{IND}, HFS V_{LHI} Dominated
- Both LFS and HFS Experience I_p gains with Increased V_{LHI}

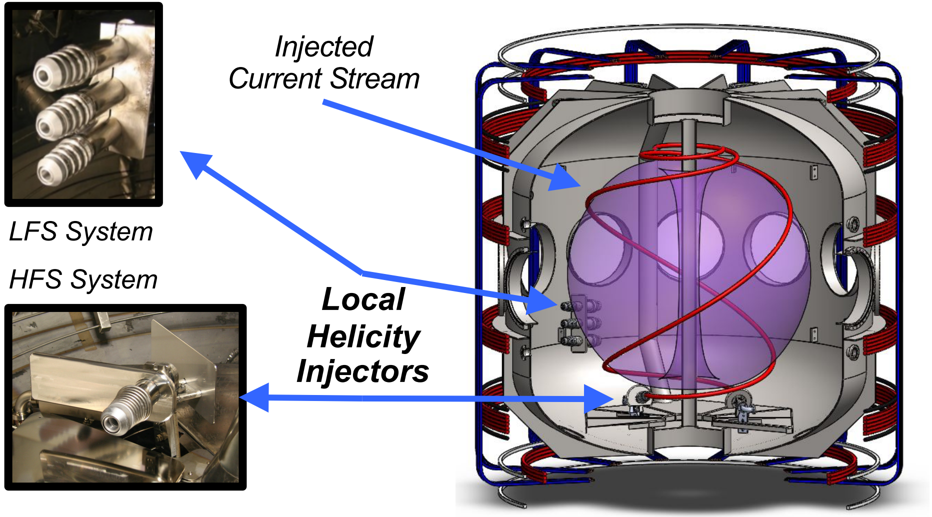
- Paths to Higher I_p
- Path to High I_p Depends on Choice of LHI Injector Geometry
- Enhanced V_{LHI} in LFS Discharges: Modest Increases in Achieved I_p
- Taylor Limit Increase Early in LFS Discharges: Access to Higher I_p
- Moving Toward I_p ~ 0.3 MA in Pegasus Enhanced

- LHI Scalability
- Model Applied to NSTX-U Geometry for Initial I_p ~ 0.8 MA Scenario
- Helicity Dissipation in LHI Plasmas is Under Active Investigation
- LHI Predictive Capability Validated on Pegasus and Applied Toward Projections for High Ip Startup

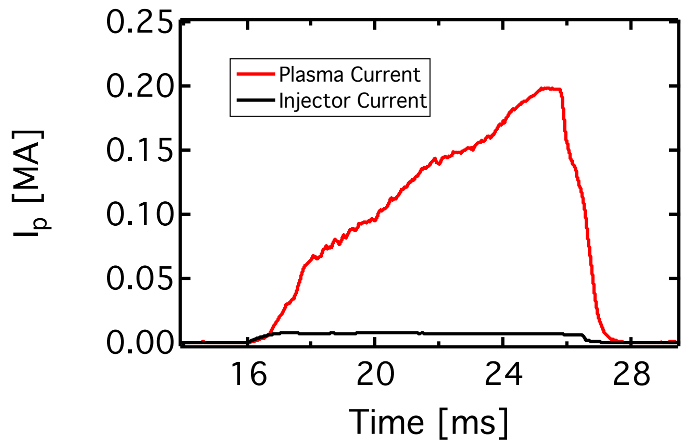
Developing Predictive Capability for Local Helicity Injection (LHI) Startup

- LHI is a promising, scalable non-solenoidal startup technique
- 0-D power-balance model developed to interpret, predict LHI I_p evolution
- Model previously shown to reproduce experimental I_p(t)
- Model provides insight into developing LHI and scaling high I_p startup
  - Understanding electron behavior important for projections to NSTX-U and beyond

LHI is a Promising Non-Solenoidal Startup Technique



Non-Solenoidal, High $I_p \leq 0.2$ MA ($I_{inj} \leq 8$ kA)

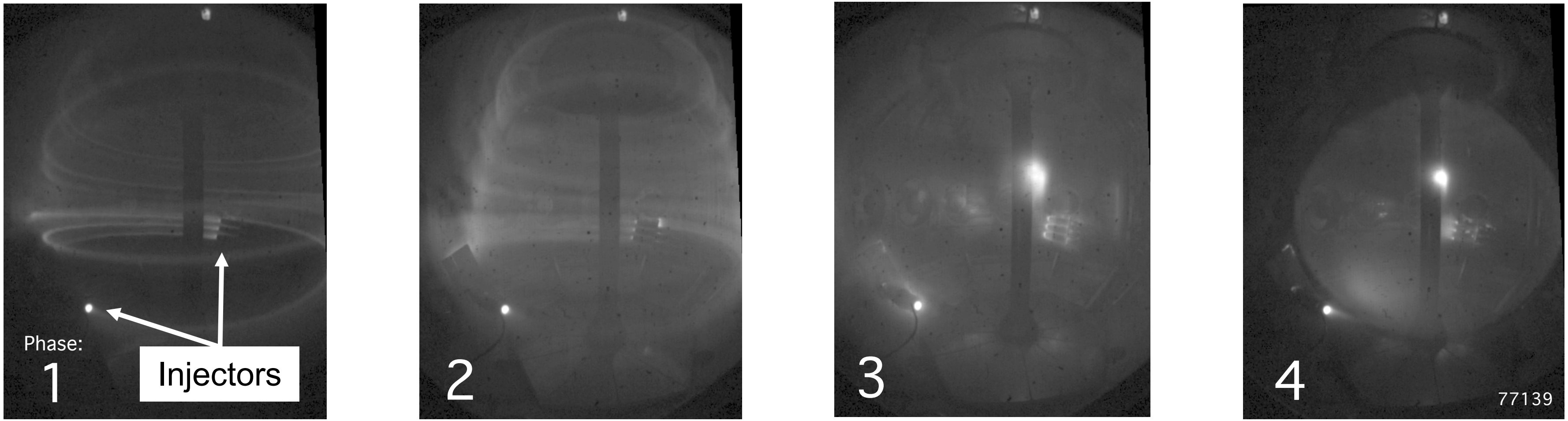


Pegasus Parameters

A	1.15 – 1.3
R [m]	0.2 – 0.45
I_p [MA]	$\leq$ 0.25
B_T [T]	< 0.15
Δt_{shot} [s]	$\leq$ 0.025

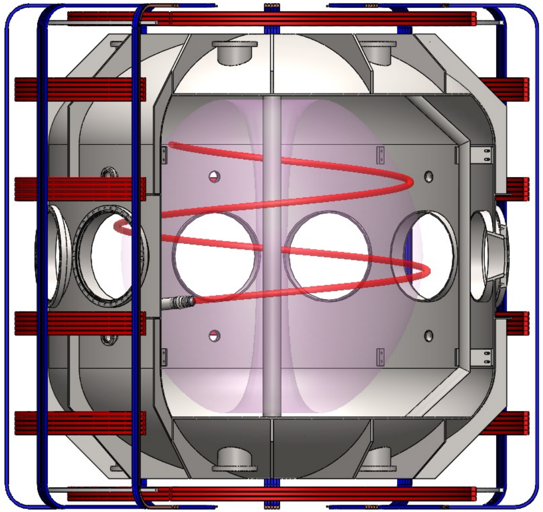
- Edge current extracted from injectors
- Relaxation to tokamak-like state via helicity-conserving instabilities
- Used routinely for startup on PEGASUS

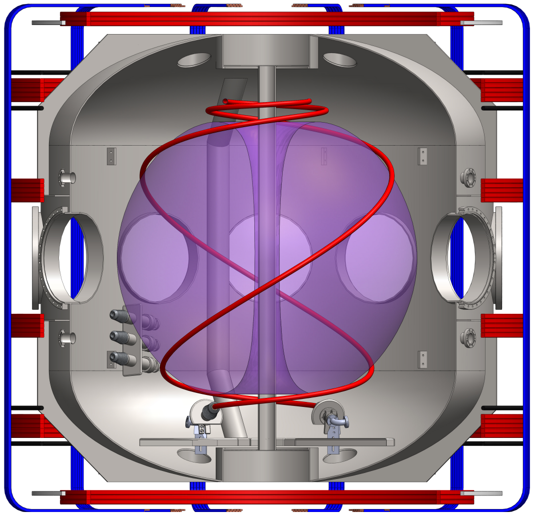
Local Plasma Sources Inject Current Streams that Reconnect to Form Tokamak-like Plasma



Local source:	Instability:	Reconnection:	Bias shutdown:
Helical plasma streams	Current driven along streams	Relaxation to tokamak-like state, current growth	High-I_p tokamak

Injector Location in LHI Emphasizes Different CD Mechanisms





Low-Field-Side (LFS) Injection:

- Injectors near outboard midplane
- Dominated by inductive drive

High-Field-Side (HFS) Injection:

- Injectors in lower divertor
- Dominated by helicity drive

I_p Evolution Predicted from Power Balance and Taylor Relaxation

0-D LHI model predicts I_p(t) based on lowest of two limits:

- Poynting's Theorem at plasma boundary sets I_p(t):

$$\underbrace{I_p V_s}_{\text{Plasma surface-voltage}} \approx \underbrace{\iiint \frac{\partial}{\partial t}\left(\frac{B_\theta^2}{2\mu_0}\right) dV}_{\text{Internal magnetic energy storage}} + \underbrace{I_p^2 R_p}_{\text{Resistive Dissipation}} - \underbrace{I_p V_{LHI}}_{\text{Non-inductive current drive (LHI)}}$$

- Taylor relaxation limit strictly enforced as maximum I_p

Global Helicity Balance, Taylor Relaxation Limit Maximum I_p

Helicity balance in a Tokamak geometry:

$$\frac{dK}{dt} = -2\int_V \eta \mathbf{J}\cdot \mathbf{B}\, d^3x - 2\frac{\partial \psi}{\partial t}\Psi - 2\int_A \Phi \mathbf{B}\cdot d\mathbf{s} \quad \Rightarrow \quad I_p \le \frac{A_p}{2\pi R_0 \langle \eta \rangle}\left(V_{ind} + V_{eff}\right)$$

- Helicity injection provides an effective loop voltage:

$$V_{LHI} \approx \frac{A_{inj} B_{\phi,inj}}{\Psi_T} V_{inj}$$

Taylor relaxation of a force-free equilibrium:

$$\nabla \times B = \mu_0 J = \lambda B$$

$$\lambda_p \le \lambda_{edge} \quad \Rightarrow \quad I_p \le \sqrt{\frac{1}{B_{\theta+V,inj}/I_p}\frac{\Psi_T I_{inj}}{2\pi R_{inj} w_{inj}}}$$

A_p	Plasma area
Ψ_T	Plasma toroidal flux
w_{inj}	Edge current channel width
$B_{\theta+V,inj}$	Poloidal field in injection region

*Eiditis et al., Journal of Fusion Energy 26, pp. 43 (2007)

Battaglia *et al.*, Nucl. Fusion **51, 073029 (2011)

0-D Power Balance Model Incorporates Analytic Plasma Inductance (ψ_{L_e}, ψ_{PF}) Formulae

$$I_p\left[V_{PF} + V_{Le} - V_{W_m} - V_{IR} + V_{NICD}\right] = 0$$

V_{IND} (= $V_{PF} + V_{Le} - V_{W_m}$)

- I_p(t) determined via numerically solving initial value problem
- Inputs: shape(t), $\langle\eta_p\rangle$, l_i(t), β_p(t)
- Analytic, finite-*A* descriptions calculate ψ_{L_e}, ψ_{PF} assuming radial force balance

Inductive Drive from Poloidal Fields

$$V_{PF} = -\frac{\partial}{\partial t}\psi_{PF} \approx -\frac{\partial}{\partial t}\left[I_p \; M_V \pi R_0^2 \left({}^{B_V}\!/\!_{I_p}\right)\right]$$

$$\frac{1}{\mu_0}\frac{B_V}{I_p} = -\frac{1}{4\pi R_0}\left\{\frac{1}{\mu_0}\frac{\partial L_e}{\partial R_0} + \left(\beta_p + \frac{l_i}{2}\right) - \frac{1}{2}\right\}$$

$$\frac{1}{\mu_0}M_V = \mathrm{R}_0\frac{(1-\epsilon)^2}{(1-\epsilon)^2 f_c + f_d\sqrt{\kappa}}$$

Plasma Magnetic Energy Change

$$V_{W_m} = \frac{1}{I_p}\frac{\partial}{\partial t}\left[\tfrac{1}{2}L_i I_p^2\right] - V_{RTT} \qquad L_i = \mu_0 l_i \frac{V_P}{{\ell_P}^2}$$

$$V_{RTT} = \frac{I_p}{2\mu_0}\oiint_{S_P}\left(\frac{B_p}{I_p}\right)^2 \vec{v_b}(\theta)\cdot\hat{n}dS$$

$\left({}^{B_p}\!/\!_{I_p}\right)$ from Miller local equilibrium model

Shape, B_t, q_{edge} dependencies

Inductive Drive from Shape Evolution

$$V_{L_e} = -\frac{\partial}{\partial t}\left[L_e I_p\right]$$

$$\frac{1}{\mu_0}L_e = R_0\frac{f_a(1-\epsilon)}{(1-\epsilon)+\kappa f_b}$$

f_a = fitting function

Resistive Dissipation

$$V_{IR} = I_p R_p = I_p\left(\frac{\langle\eta_p\rangle 2\pi R_0}{A_p}\right)$$

LHI Drive

$$V_{LHI} = \frac{A_{inj}B_{\phi,inj}}{\Psi_T}V_{inj}$$

* *S.P. Hirshman and G.H. Nielson 1986 Phys. Fluids* ***29*** *790*

** O. Mitarai and Y. Takase 2003 *Fusion Sci. Technol.*

S. Ejima et al 1982 Nucl. Fusion ***22*** *1313*

J.A. Romero and JET-EFDA Contributors 2010 Nucl. Fusion ***50*** *115002*

Analytic Plasma Inductance Formulas Expanded, Re-Calibrated for Non-uniform B_V, Ultralow-A

- L_e, M_V coefficients refit to relevant equilibria:
  - Generated using KFIT (> 300)
  - Spans: $1.15 \leq A \leq 8$, $1 \leq \kappa \leq 3$, $0.2 \leq l_i \leq 0.75$, $0 \leq \beta_p \leq 1$
  - Non-uniform B_V (Pegasus PF coils)

a_n (N_a=4)	b_n (N_b=4)	c_n (N_c=3)	d_n (N_d=3)	e_n (N_e=4)
1.438	0.149	-0.293	0.003	0.080
2.139	1.068	-0.349	0.334	-0.260
9.387	-6.216	0.098	-2.018	-0.267
-1.939	4.126			1.135

- l_i, β_p dependences added to L_e:

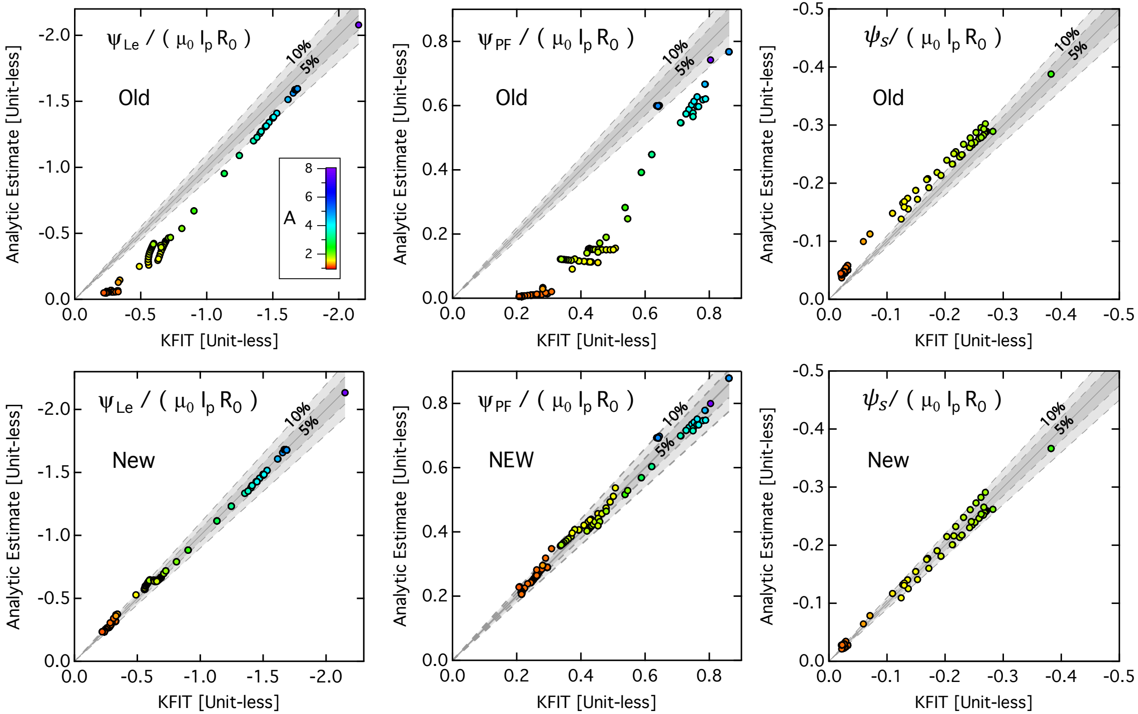
$$f_{a,new} = \left\{1 + \sum_{n=1}^{\frac{N_a}{2}} a_n \left(\sqrt{\epsilon}\right)^n\right\} \ln\left(\frac{8}{\epsilon}\right) - \left\{2 + \sum_{n=1}^{\frac{N_a}{2}} a_{\left(\frac{N_a}{2}+n\right)} \left(\sqrt{\epsilon}\right)^n\right\} + \left(\beta_p + \frac{l_i}{2}\right) \sum_{n=1}^{N_e} e_n \left(\sqrt{\epsilon}\right)^n$$

$$f_{g,new} = -\frac{1}{\epsilon} + \ln\left(\frac{8}{\epsilon}\right) \sum_{n=1}^{\frac{N_a}{2}} \frac{\mathrm{n}}{2} a_n \left(\sqrt{\epsilon}\right)^{n-2} - \sum_{n=1}^{\frac{N_a}{2}} \left(a_n + \frac{n}{2} a_{\left(\frac{N_a}{2}+n\right)}\right) \left(\sqrt{\epsilon}\right)^{n-2} + \left(\beta_p + \frac{l_i}{2}\right) \sum_{n=1}^{N_e} \frac{n}{2} e_n \left(\sqrt{\epsilon}\right)^{n-1}$$

J.L. Barr (to be submitted to Nucl. Fusion 2017)

Equilibrium-Calibrated Inductance Model Improves Estimates of Non-Solenoidal V_{IND}

- Maintaining radial force balance provides V_{IND}
  - Originally calculated via H-N formulae
- Old ψ_S, ψ_{L_e}, ψ_{PF} show error at A<2 in Pegasus equilibria
  - $\psi_S = \psi_{L_e} + \psi_{PF}$
  - Led to incorrect drive contributions
- Revised V_{IND} model developed
  - Added l_i, β_p to f_a in L_e formulation
  - Derived new fit coefficients based on fit to Pegasus Equilibrium database
- ψ_S, ψ_{L_e}, ψ_{PF} accurate to within 10% using revised formulas

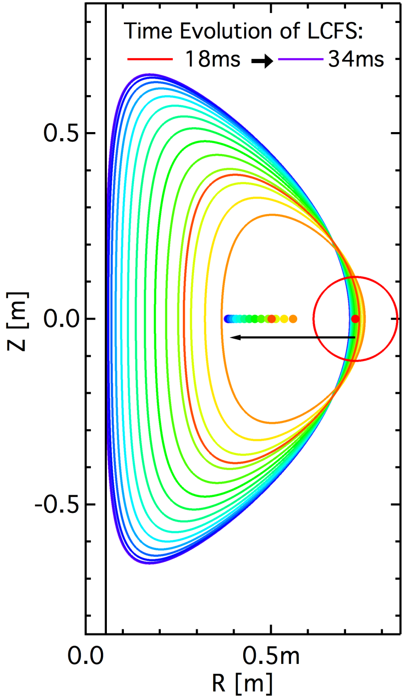


S.P. Hirshman and G.H. Nielson 1986 Phys. Fluids **29** 790
J.L. Barr (to be submitted to Nucl. Fusion 2017)

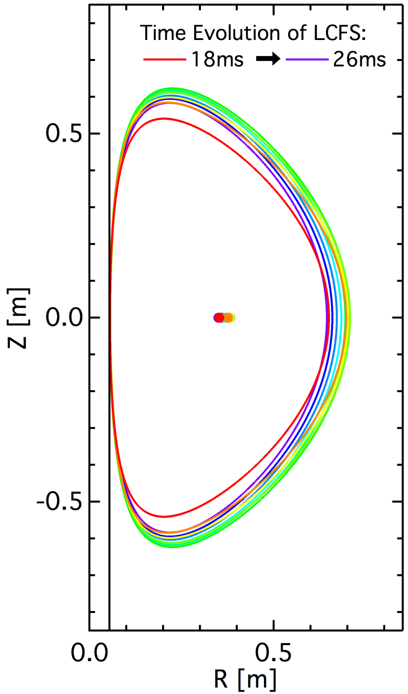
0-D Model Takes Plasma, Injector Parameters as Inputs

- Shape(t)
  - $R_0(t)$, $a(t)$, $\kappa(t)$, $\delta(t)$
  - Vertical symmetry
- $<\eta>(t)$, $l_i(t)$, $\beta_p(t)$
  - Constant $<\eta>$ assumed
    - Neoclassical modified Spitzer
    - $\eta_{neo}/\eta_{Spitzer}$~2-4x
    - Adjusted to match I_p
  - $\beta_p = 0$
  - ℓ_i dropping: 0.5 → 0.2
- Injector Inputs:
  - A_{inj}, R_{inj}, $V_{inj}(t)$, $I_{inj}(t)$

Typical LFS Geometry Evolution

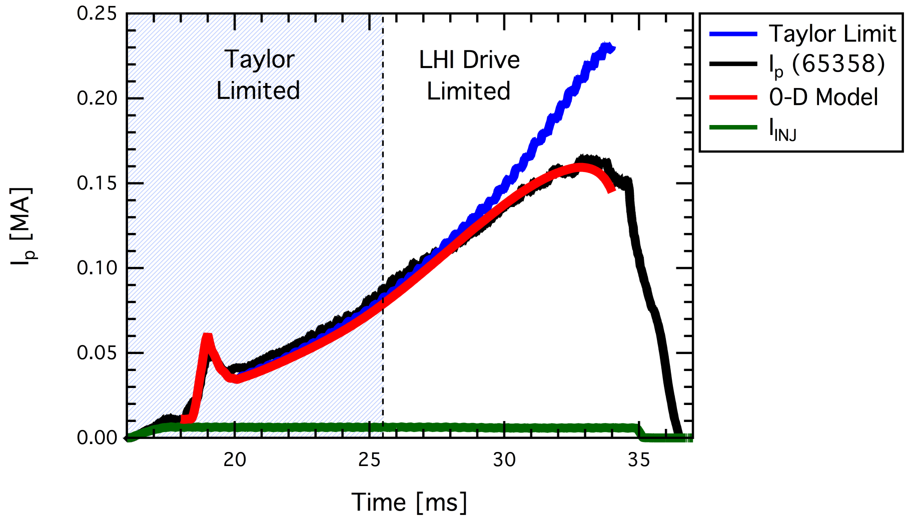


Typical HFS Geometry Evolution

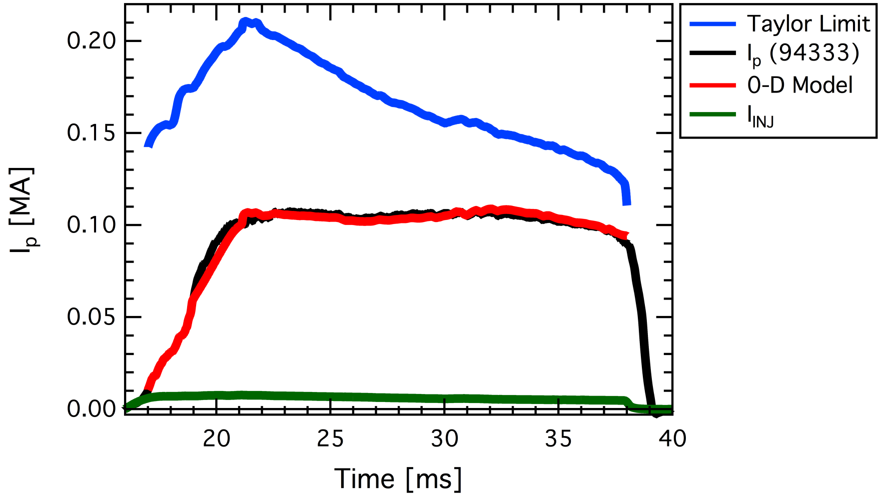


LFS Exhibit Taylor Limit Phase, HFS V_{LHI} Limited Through Entire Discharge

- LFS
  - Taylor-limited early in discharge



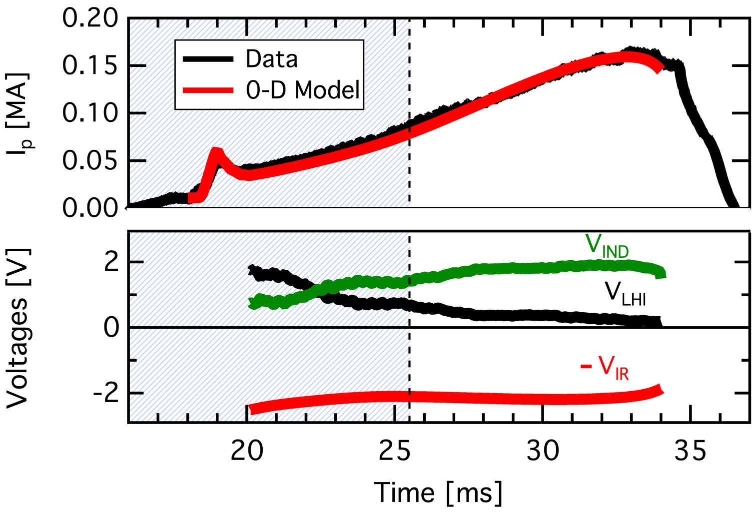
- HFS
  - Drive limited throughout discharge

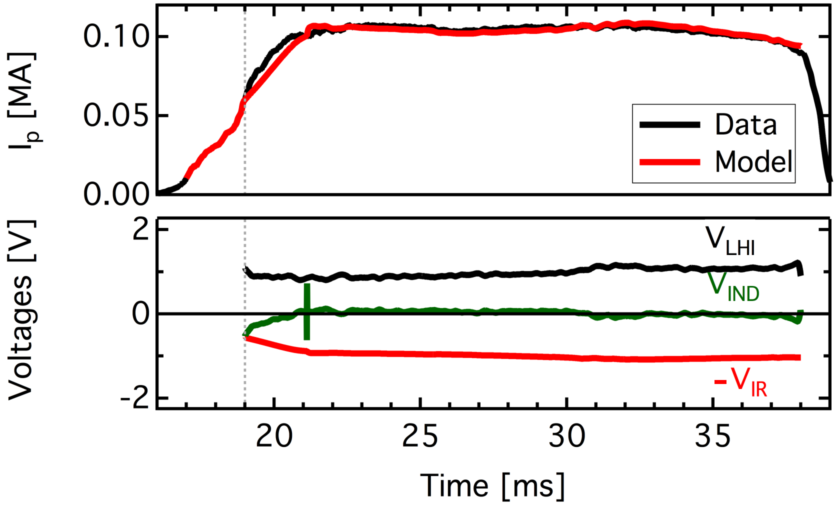


LFS Discharges Driven Primarily by V_{IND}, HFS V_{LHI} Dominated

- LFS
  - Inward expansion ➞ large V_{IND}
  - V_{LHI} drops as plasma grows to large size, low-A

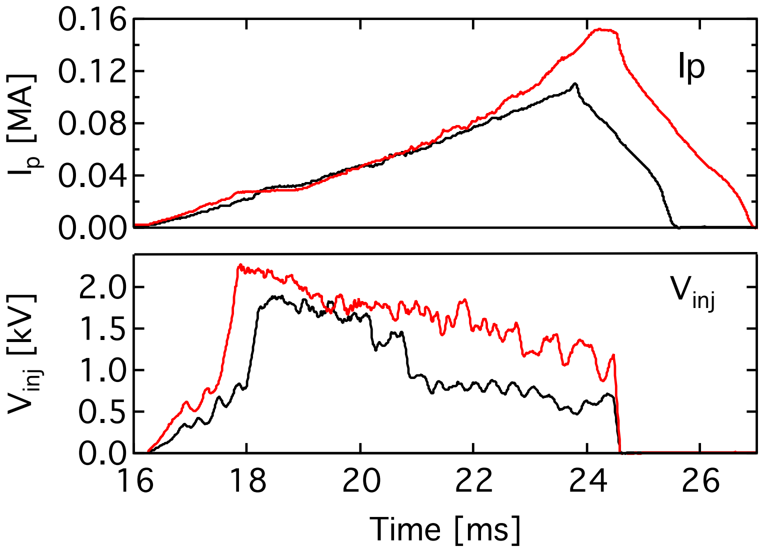
- HFS
  - Shape ~ constant ➞ minimize V_{IND}
  - V_{LHI} ~ constant, sustains discharge



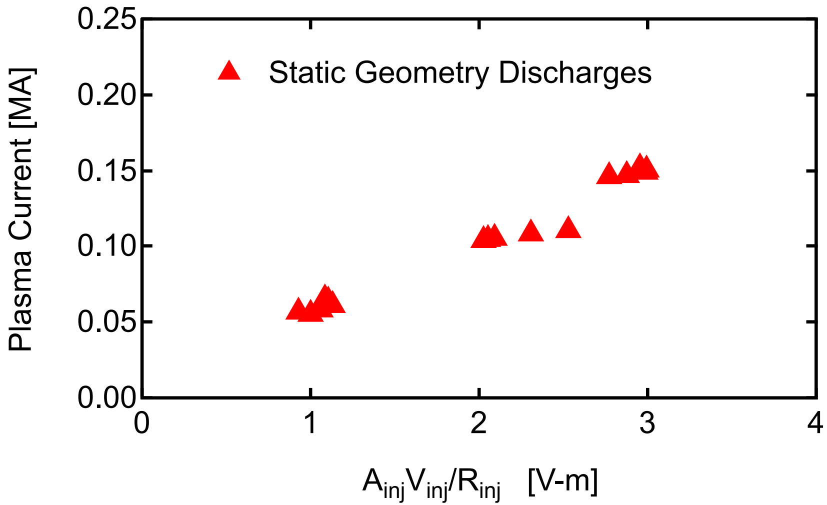


Both LFS and HFS Experience I_p gains with increased V_{LHI}

- LFS
  - I_p increases with experimental V_{LHI}
  - Inductive effects still dominant



- HFS
  - Static geometry → V_{LHI} dominant
  - Linear increase in I_p with experimental V_{LHI}



Path to High I_p Depends on Choice of LHI Injector Geometry

LFS

- Increase Taylor limit in first half of discharge
  - Increase $I_{TF}, \frac{I_{INJ}}{w_{INJ}}$
- Increase A_{INJ} while lowering V_{INJ}
  - Location of injectors allows for larger Injector Area
  - Increase A_{INJ} → enables lower V_{INJ} → Mitigate PMI
- Plasma position and shape control challenges
  - Maintain coupling to guns
  - Control geometry evolution for inductive drive

HFS

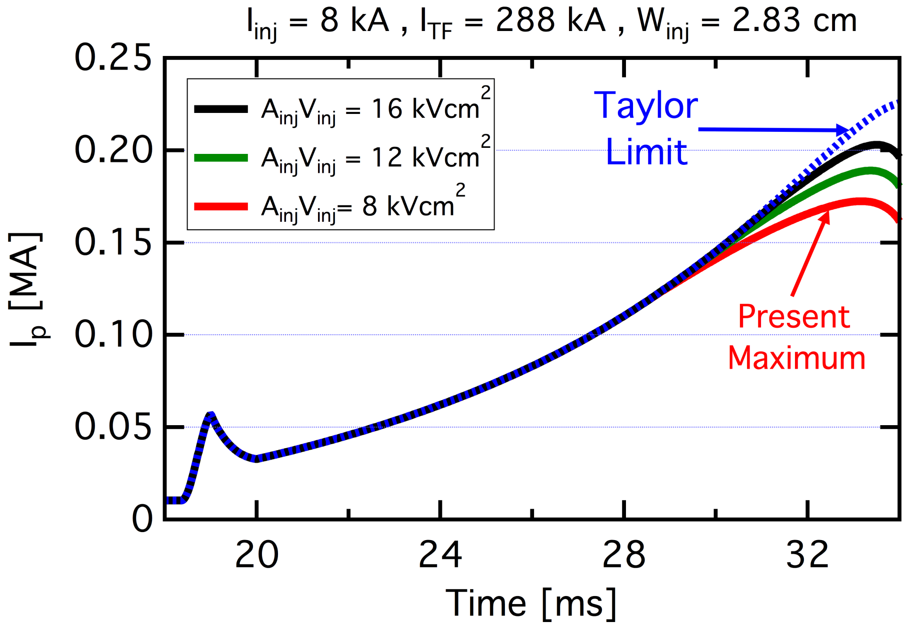
- Decrease R_{INJ}
  - $V_{LHI} \sim \frac{1}{R_{INJ}}$
- Increase A_{INJ}, V_{INJ}
  - Possible engineering challenges
  - Increased V_{INJ} increases PMI
- Improved performance at increased TF
- $V_{LHI}(t)$ control for $I_p(t)$ path optimization

CONCLUSION: Overall, modelling suggests largest gains in I_p accessible using LFS LHI

Enhanced V_{LHI} in LFS Discharges: Modest Increases in Achieved I_p

- Increase V_{LHI}
  - Increase effective A_{inj}
    - Larger gun cross-section
    - More guns
  - Increase V_{inj}
    - Engineering and PMI limitations
- Modest LFS performance gains with increased V_{LHI}
  - Taylor-limited phase inhibits I_p growth

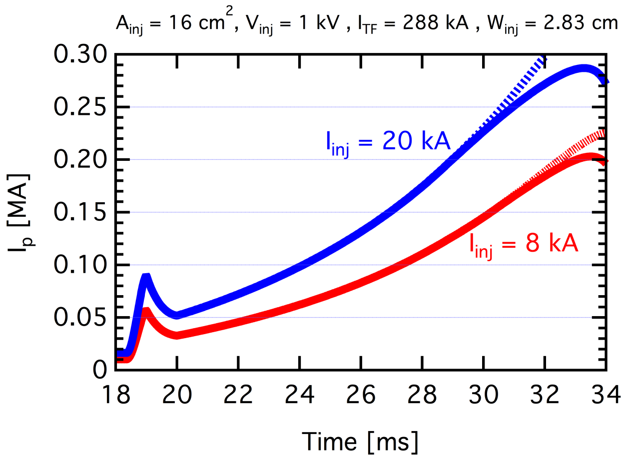
Maintain Taylor Limit → Increasing V_{LHI}



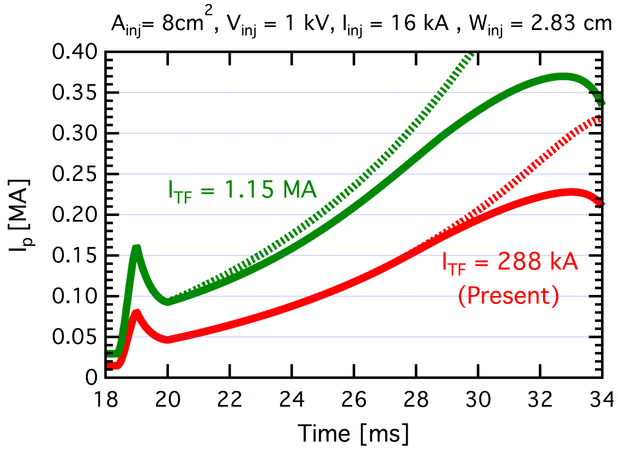
Taylor Limit Increase Early in LFS Discharges: Access to Higher I_p

- LFS discharges experience an extended Taylor-limited I_p phase
- Increase Taylor limit through injector design or facility modifications $I_{TL} \sim \sqrt{\frac{I_{TF} I_{INJ}}{W_{INJ}}}$

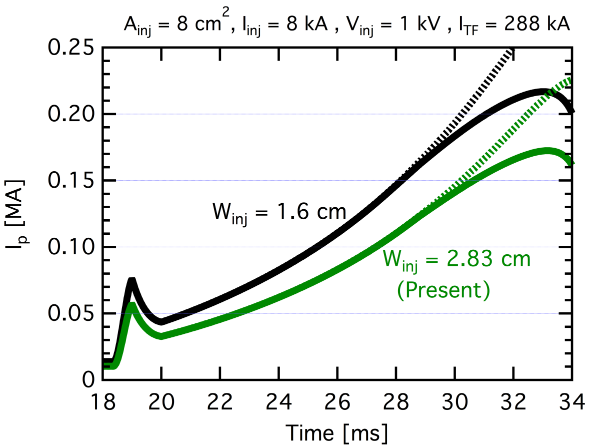
Increase I_{inj}



Increase I_{TF}



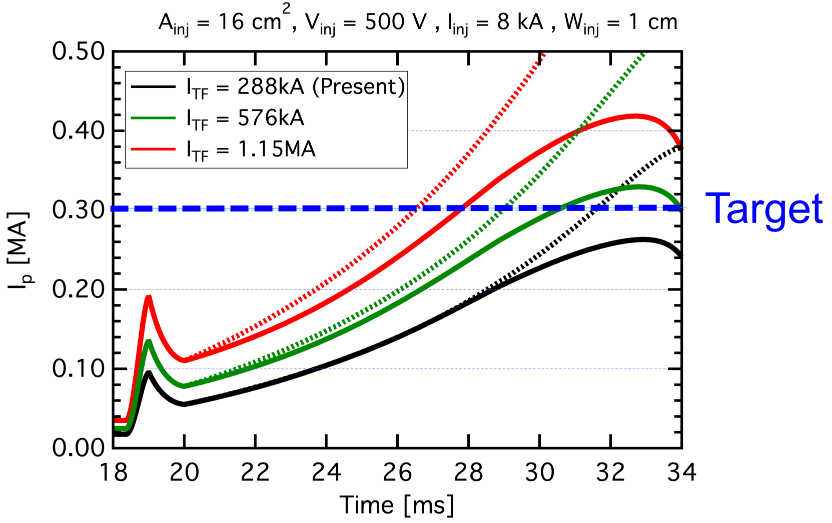
Decrease W_{inj}



Moving toward $I_p \sim 0.3$ MA in Pegasus Enhanced

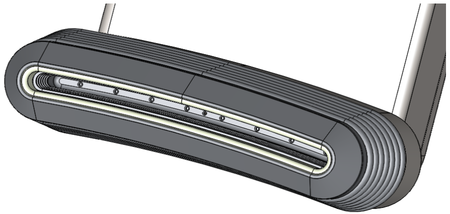
- **Testing scalable performance**
  - Large A_{inj}
  - Low V_{INJ} → Avoid PMI
  - High Taylor Limit (High I_{TF}, High $\frac{I_{INJ}}{W_{INJ}}$)
  - Unified physical structure
  - New 32 MV-A programmable voltage power supply
- **Present design concept**
  - "Slit" injector design on LFS
  - Upgrade I_{TF} (4x present level)
  - Voltage controlled power supplies
  - Nominal PF limit $I_p \sim 0.3$ MA
  - Design overhead for physics and engineering margins

Model projection for next gen. LHI system



Non-circular, High-A_{inj} Helicity Injector Rendering

High $A_{inj} = 16\ cm^2$
Low $W_{inj} = 1$ cm aperture



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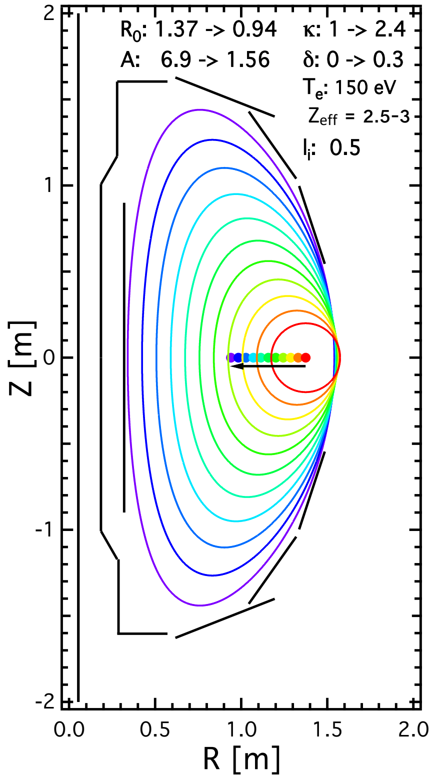
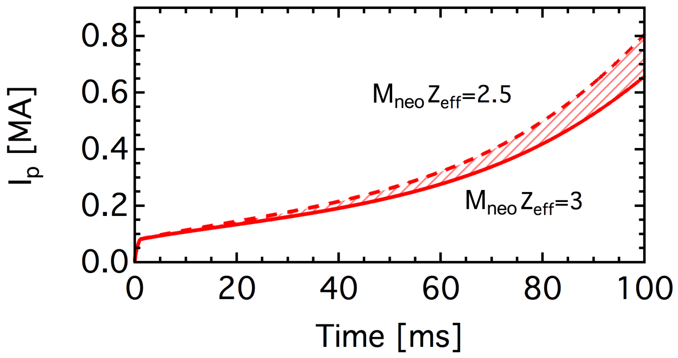
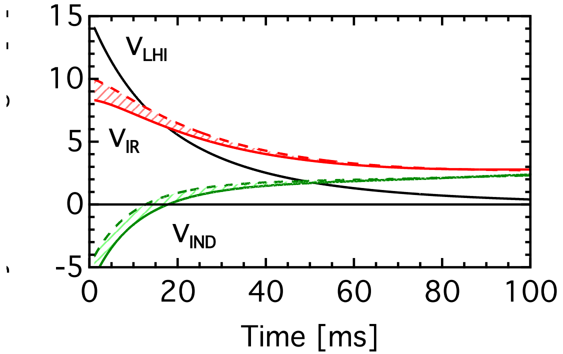
Model Applied to NSTX-U Geometry for Initial $I_p \sim 0.8$ MA Scenario







- $I_p \sim 0.8$ MA appears achievable with reasonable LHI system
  - $\langle T_e \rangle \sim 150$ eV assumed; $Z_{eff} = 2.5$–3
  - $A_{inj}V_{inj} = 20\ \mathrm{kVcm^2}$; $I_{inj} = 20$ kA
  - Improvements at higher TF, T_e could lower injector requirements

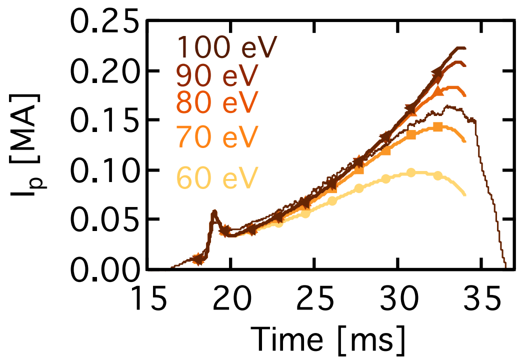
- Important physics questions remain for projection:
  - Confinement scaling to large size, high-B
  - Current drive effectiveness at high-B

*C. Neumeyer *et all* (2009) *23rd IEEE/NPSS Symposium on Fusion Engineering*
**C. Neumeyer (2001) "NSTX Internal Hardware Dimensions":
http://nstx.pppl.gov/nstx/Engineering/NSTX_Eng_Site/Technical/Machine/NSTX_Engr_Machine_Dims_cm.html

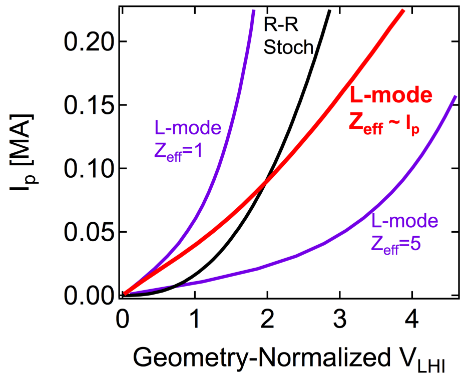
Helicity Dissipation in LHI Plasmas is Under Active Investigation

- Model strongly dependent on assumed resistivity $\eta_{neo} = Z_{eff} M_{neo} \eta_{spitzer}$
  - Constant $Z_{eff} M_{neo}$ ➞ $\langle T_e \rangle_{vol} \sim 73$ eV
  - Consistent with T_e from Thomson scattering
- Electron behavior and impurity studies are an active focus
  - Z_{eff}, radiation losses, profile effects
- Diagnostic improvements to study effects
  - SPRED upgrade
  - VB spectroscopy system improvements
  - New bolometer
  - Thomson upgrade

Model sensitivity to resistive dissipation



Confinement projections for varying Z_{eff}



C. Rodrieguez Sanchez *Poster* UP11.00089
***G.M. Bodner Poster* UP11.00087**

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LHI Predictive Capability Validated on Pegasus and Applied Toward Projections for High I_p Startup

- Different LHI injector geometries exhibit different dominant drive mechanism
  - LFS: V_{IND} dominated due to inward expansion
  - HFS: V_{LHI} dominated, V_{IND} minimized from static geometry

- Paths to high I_p depend on injector geometry
  - LFS: Increasing Taylor limit early in discharge
  - HFS: Increase V_{LHI}

- New injector designs based on insight from power balance model
  - Modelling suggests largest gains in I_p accessible through LFS LHI

- $I_p \sim$ 0.8 MA LHI startup on NSTX-U feasible with conservative confinement assumptions
  - Physics related to helicity dissipation currently under investigation

- Work Supported by US DOE grants DE-FG02-96ER54375 and DE-SC0006928

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http://pegasus.ep.wisc.edu/Technical_Reports

A Power-Balance Model for LHI Startup

Comparison Between Different LHI Systems in Pegasus

Paths to Higher I_p

LHI Scalability