

# Physics of the Current Injection Process in Localized Helicity Injection

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PEGASUS  
Toroidal Experiment



# Local Helicity Injection (LHI) is a Promising Non-Solenoidal Tokamak Startup Technique

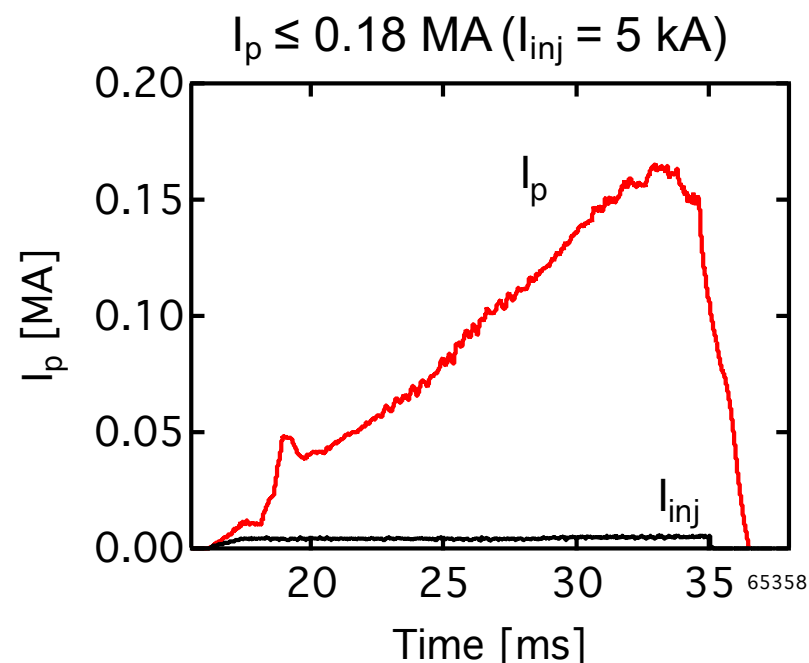
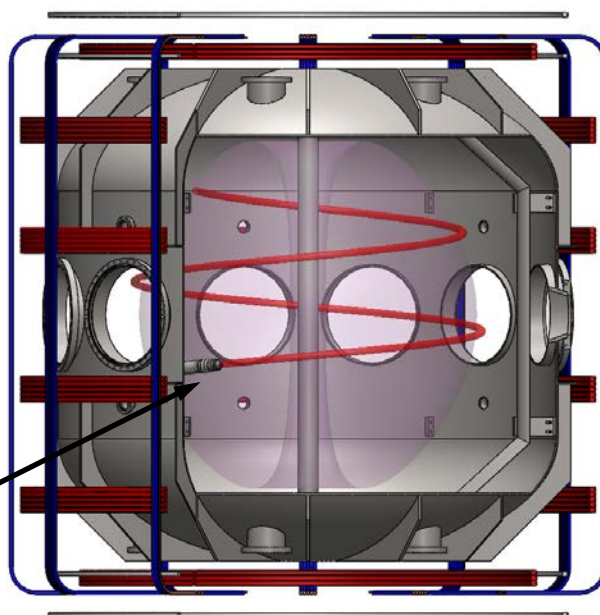
Current injected in plasma edge region



Plasma startup and current growth



Local Helicity Injectors



- Unstable current streams form tokamak-like state via Taylor relaxation
- Appears scalable to MA-class startup



# Injector Impedance Relates Key Figures of Merit

- The simple model for helicity injection posits two  $I_p$  limits related to the injector circuit

- Helicity balance limit depends on injector voltage  $V_{inj}$ :

$$I_p \leq \frac{A_p}{2\pi R_0 \langle \eta \rangle} \left( V_{ind} + \frac{A_{INJ} B_{\phi, inj}}{\Psi_T} V_{inj} \right)$$

- Taylor relaxation limit depends on current  $I_{inj}$ :

$$I_p \leq f(\epsilon, \delta, \kappa) \sqrt{\frac{\kappa A_p I_{TF} I_{inj}}{2\pi R_0 w}}$$

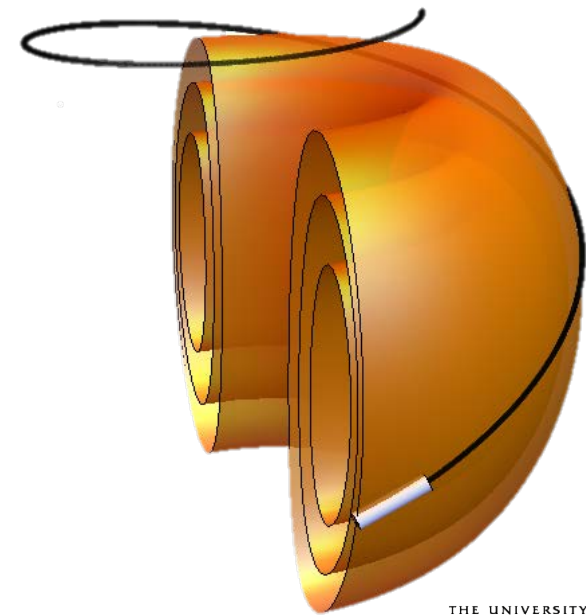
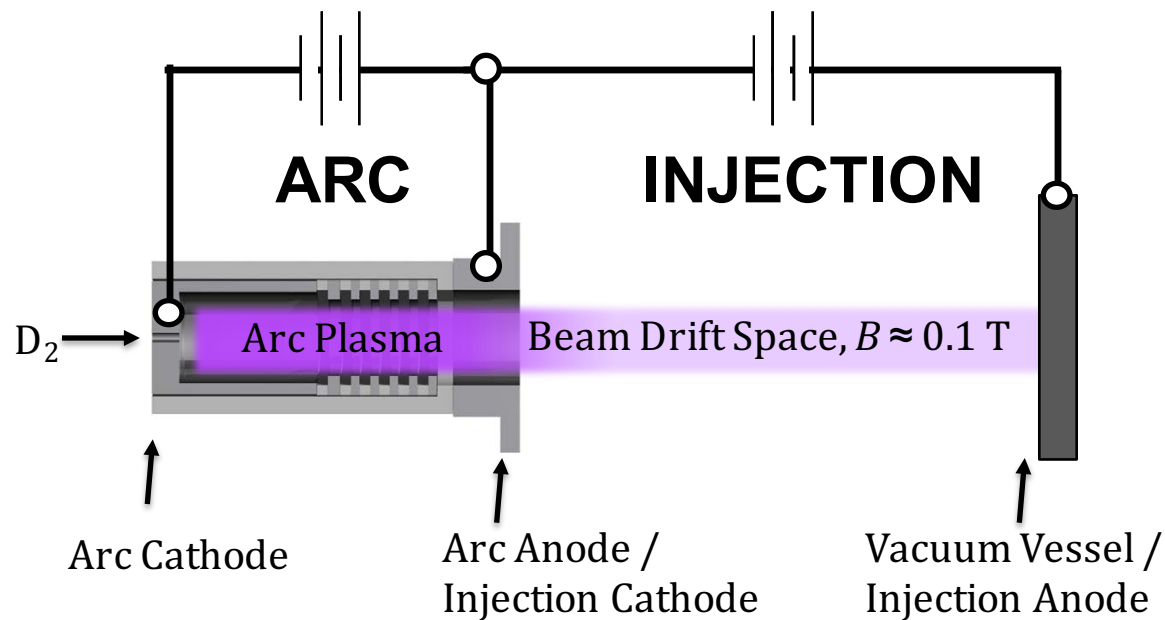
- Injector impedance describes relationship b/w  $I_{inj}$  and  $V_{inj}$

- Determined by plasma physics
- Determines feasibility and power requirements for scaling up LHI



# Edge Current Injection is Straightforward Conceptually

- Injection in Pegasus accomplished with arc plasma guns
- 2-stage circuit: bias and arc circuits “daisy chained”
- Voltage ( $\sim 1$  kV) is determined by plasma physics



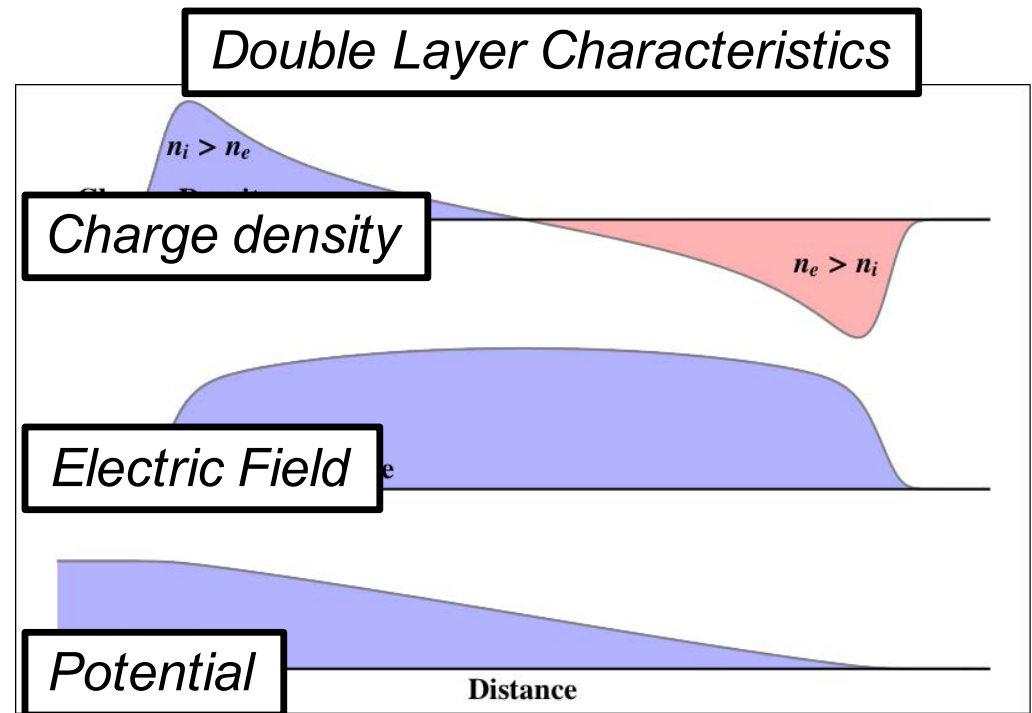


# Double Layer Sheath: A Promising Framework for Understanding Injector Impedance

- Two space-charged layers “sandwiched” to each other
- Width of space charge set by plasma, order  $\lambda_{De}$

$$J = \frac{4}{9} 1.865 \epsilon_0 \left( 1 + \sqrt{\frac{m_e}{m_i}} \right) \left( \frac{2e}{m_e} \right)^{\frac{1}{2}} \frac{V^{\frac{3}{2}}}{\ell_{DL}^2}$$

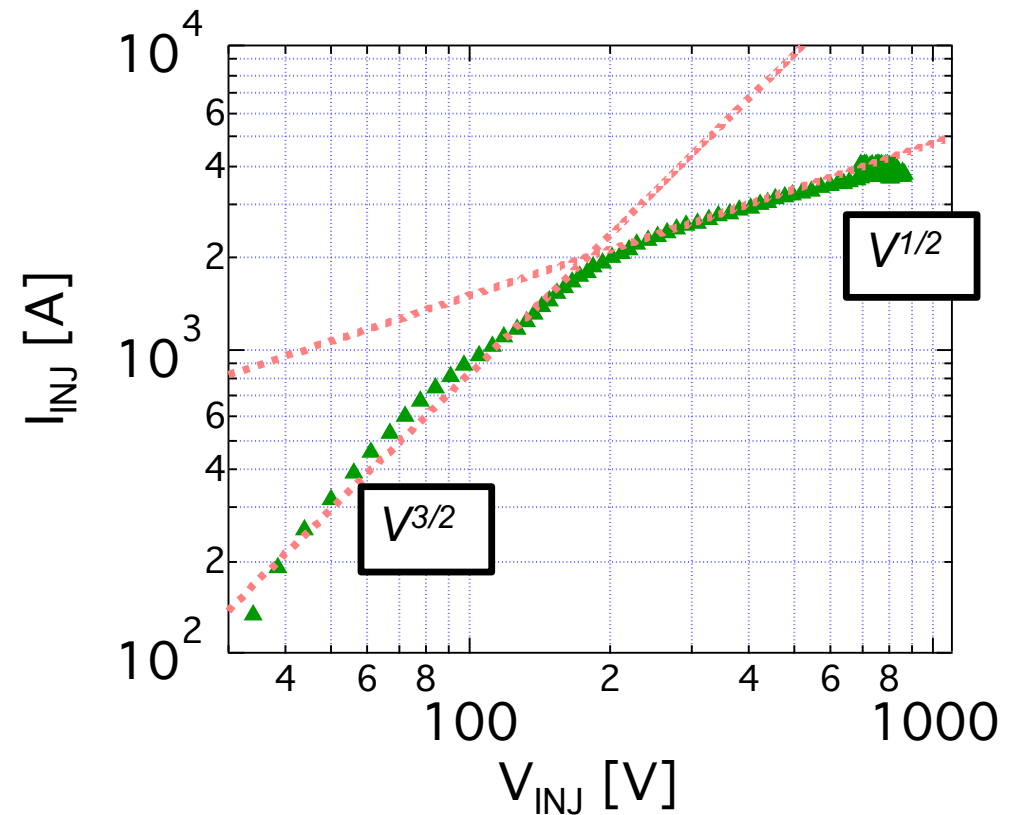
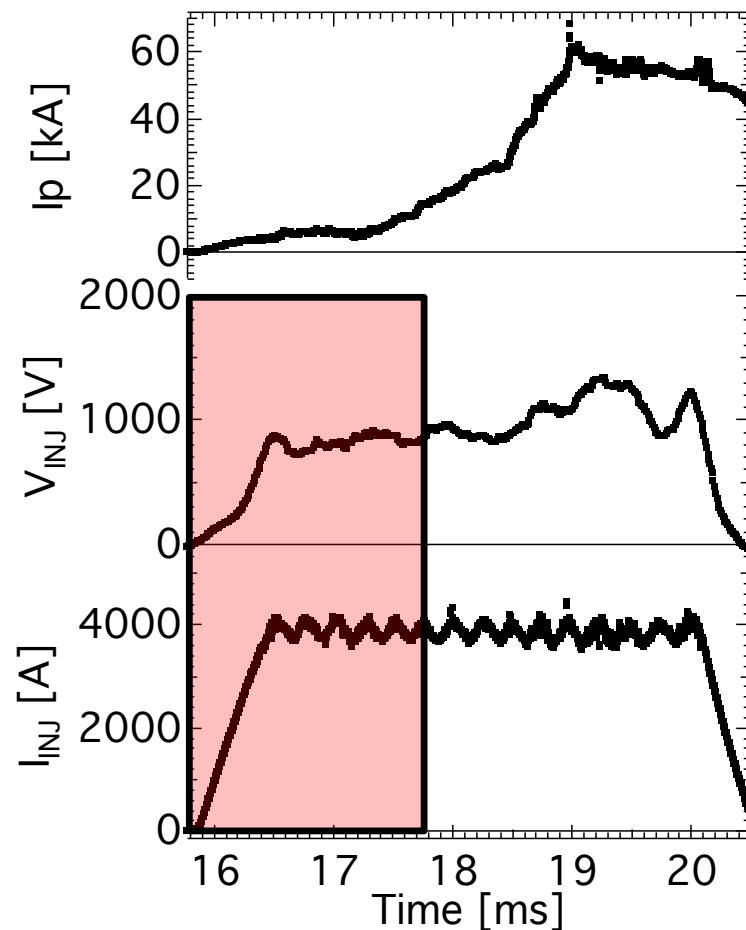
$$\ell_{DL}^2 = (\lambda_{De} \cdot \chi)^2 \Rightarrow \boxed{I \sim n_{DL} V^{\frac{3}{2}}}$$





# I-V Characteristics Show 2 Regimes

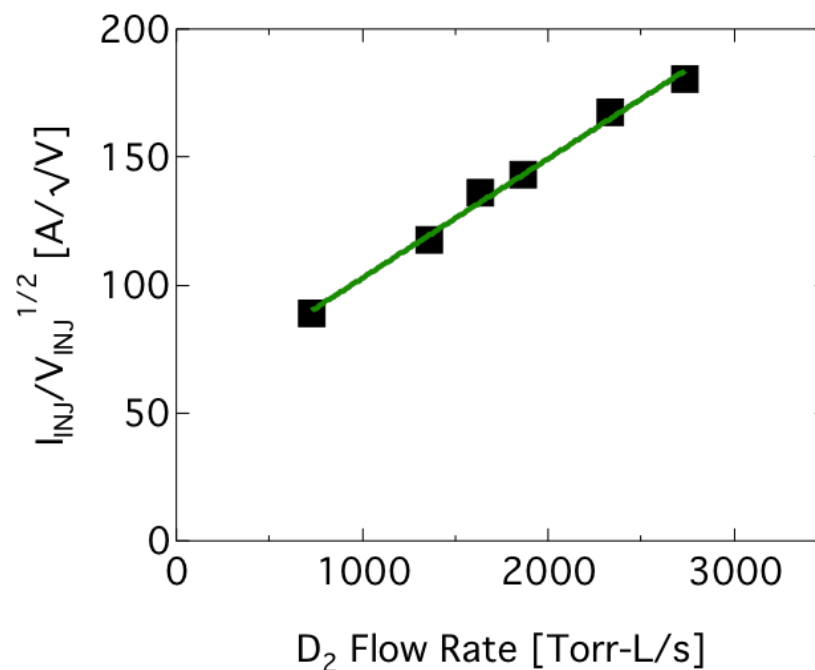
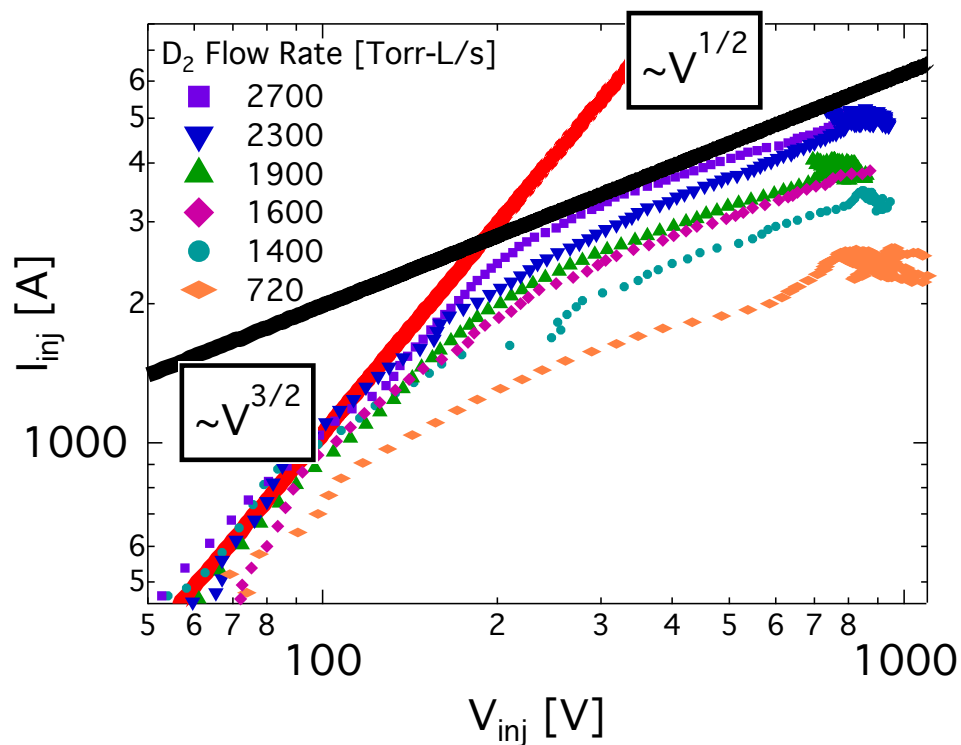
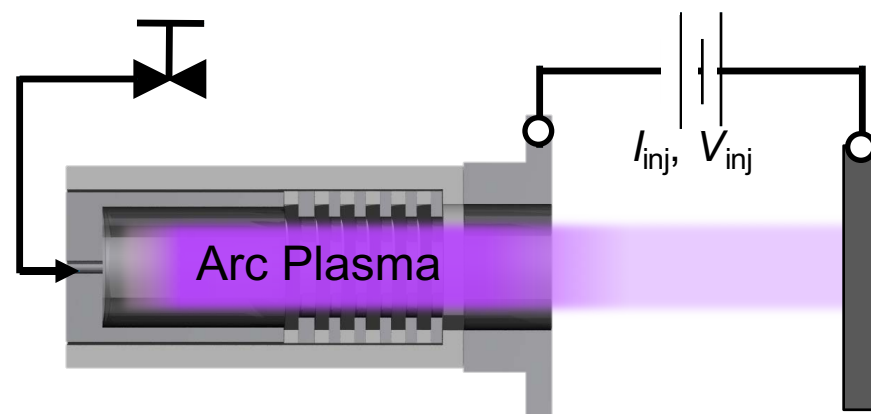
- Typical I-V relationship shows two power law regimes,  $I_{inj} \sim V_{inj}^{3/2}$ ,  $V_{inj}^{1/2}$





# Injector Impedance Has Fueling Dependence

- Deuterium gas flow rate into the source plasma scanned
- $I_{inj}/V^{1/2}$  increases with gas flow

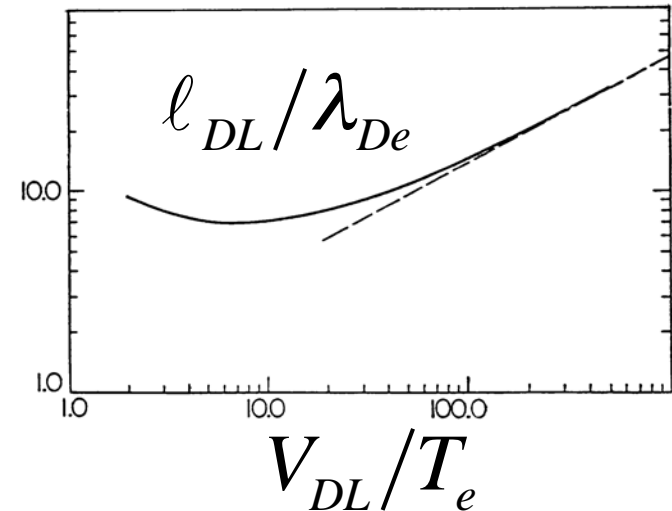




# 1: Expanding Double Layer Yields $I \sim n_{DL} V^{1/2}$

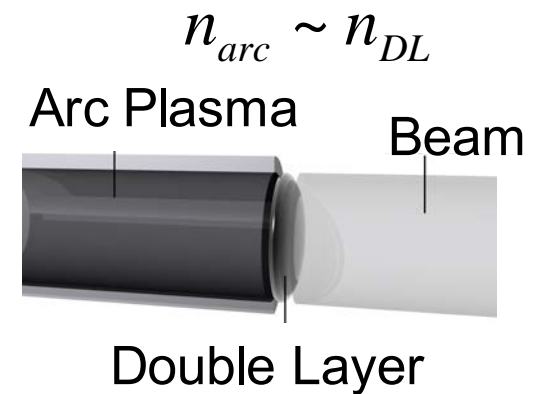
- Simulations\* find when  $V_{DL}/T_e \gg 1$ :

$$\ell_{DL} \sim \lambda_{De} \sqrt{V_{DL}/T_e}$$



Expected I-V relation is:

$$I_{inj} \sim n_{arc} \sqrt{V_{inj}}$$







## 2: Beam Neutralization Yields $I_{inj} \sim n_{edge} V_{inj}^{1/2}$

- Electron beam propagation requires drift space ions neutralize electrons:  $n_b \leq n_i$
- Typical beam values imply beam density  $n_b \sim 10^{18} \text{m}^{-3}$  - comparable to edge density  $n_{edge}$ !
- Assume drift space has same density as edge:  $n_i \approx n_{edge}$

$$I_{inj} = n_b e v_e A_{inj} \leq n_i e \sqrt{\frac{2eV_{inj}}{m_e}} A_{inj} \sim n_{edge} \sqrt{V_{inj}}$$

$$I_{inj} \sim n_{edge} \sqrt{V_{inj}}$$



# Impedance Model Based on Density Limits Created

Minimum of both limits is applicable:

Sheath expansion:

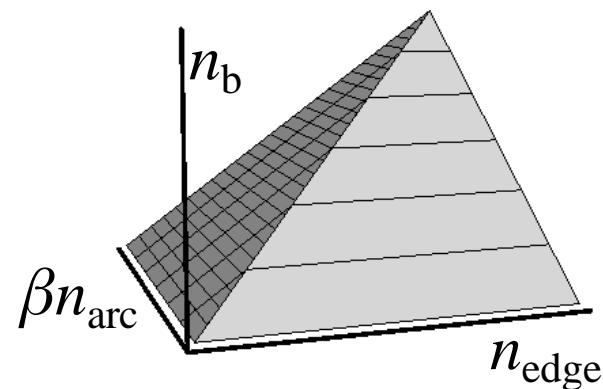
$$I_{inj} \sim n_{arc} \sqrt{V_{inj}}$$

Quasineutrality:

$$I_{inj} \sim n_{edge} \sqrt{V_{inj}}$$

Impedance Model:

$$I_{inj} = \text{Min}[n_{edge}, \beta n_{arc}] e \sqrt{\frac{2eV_{inj}}{m_e}} A_{inj}$$





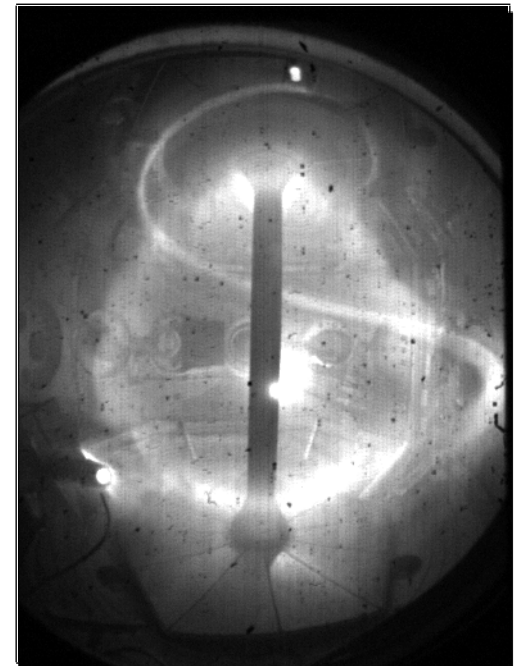
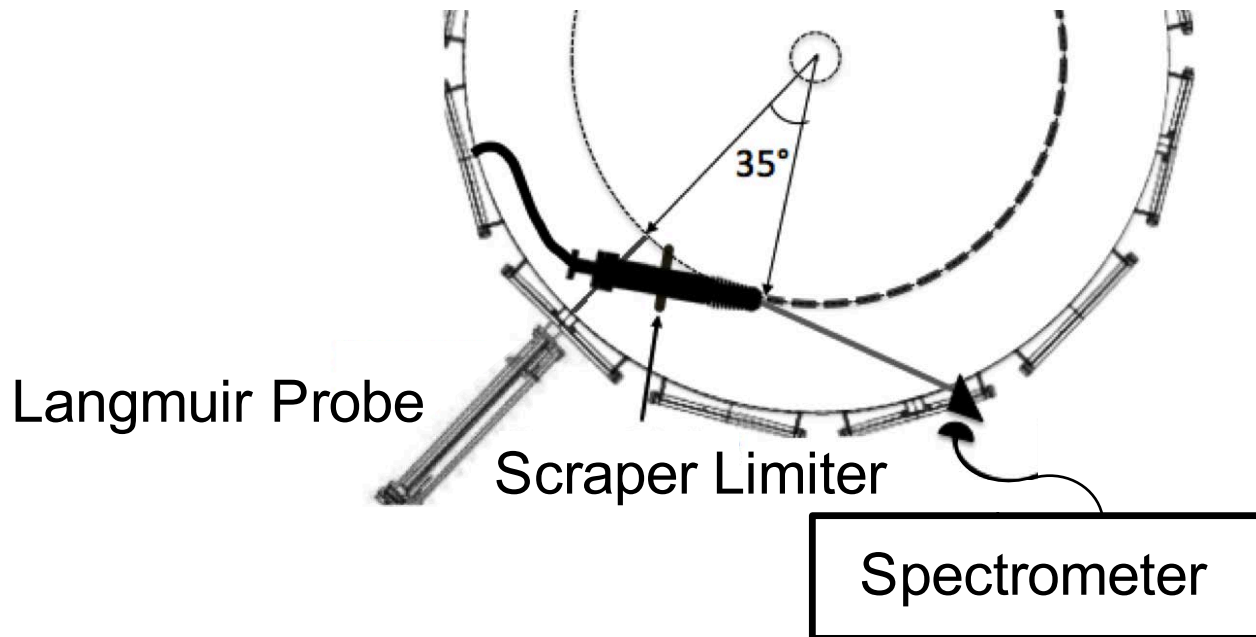
# Ohmic Plasmas Created to Test Model via Measured $n_{\text{edge}}$ , $n_{\text{arc}}$

$n_{\text{edge}}$ :

- Measured with Langmuir probe behind injector limiter
- Controlled with edge fueling

$n_{\text{arc}}$ :

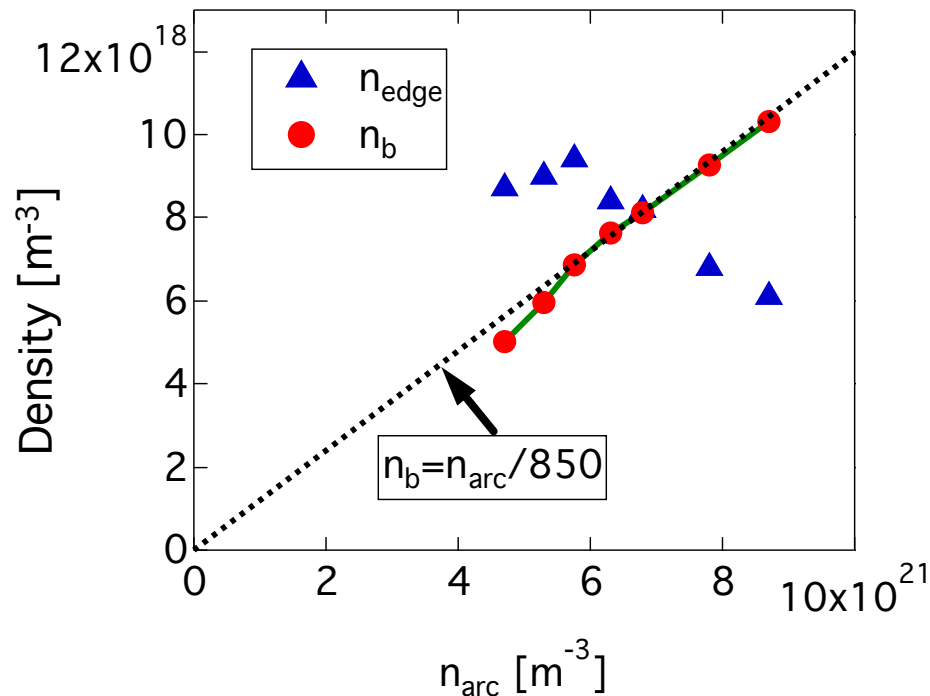
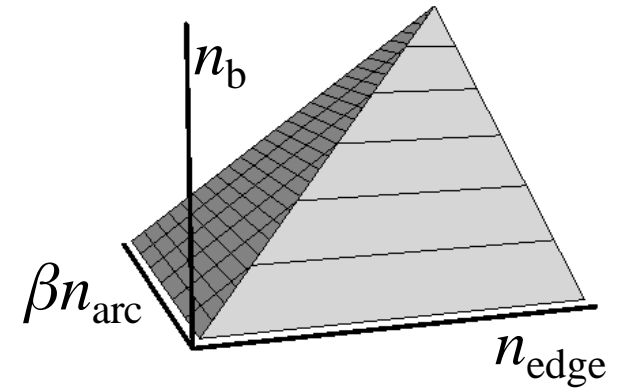
- Measured via Stark broadening of H- $\delta$  in arc channel
- Controlled with injector fueling





# $n_{\text{arc}}$ Scanned

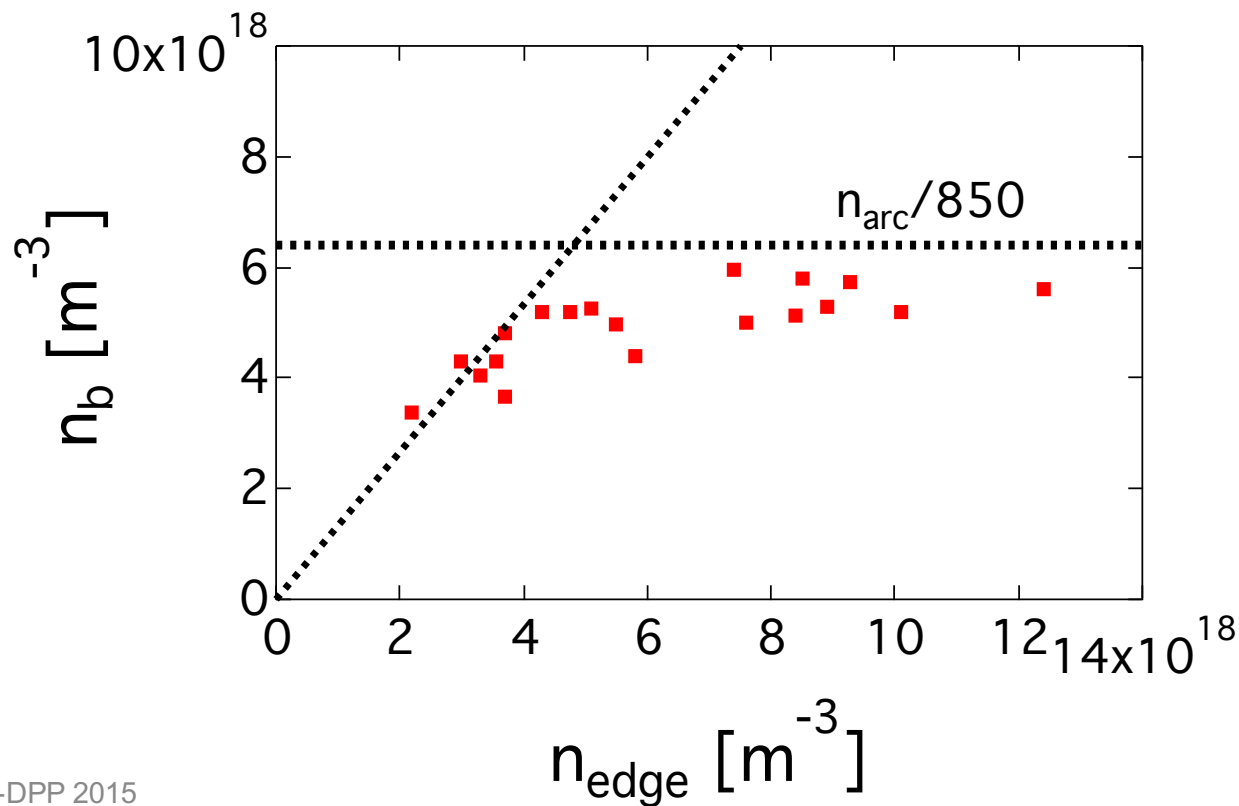
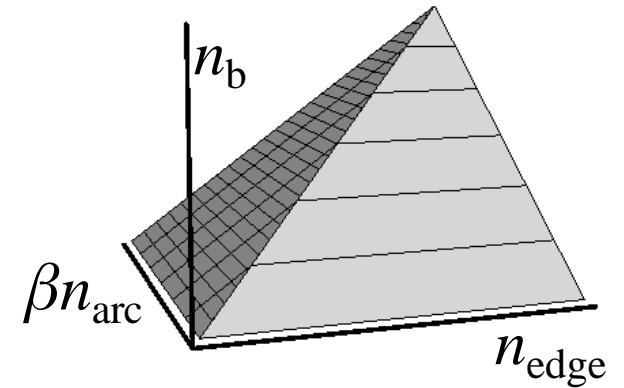
- Arc density  $n_{\text{arc}}$  scanned
- Entire scan consistent with sheath expansion:  $n_b \sim \beta n_{\text{arc}}$  where  $\beta = 1/850$





# $n_{\text{edge}}$ Scanned at constant $n_{\text{arc}}$

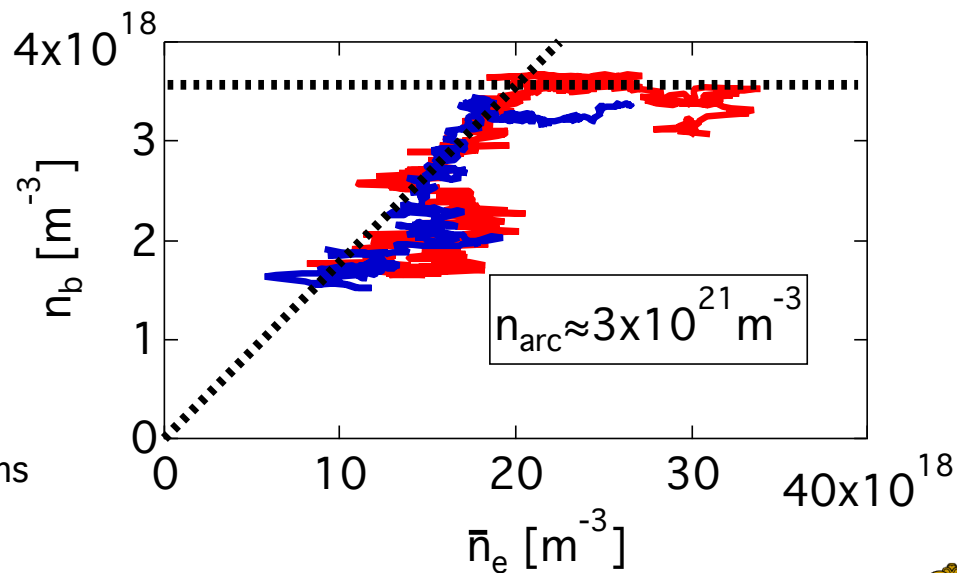
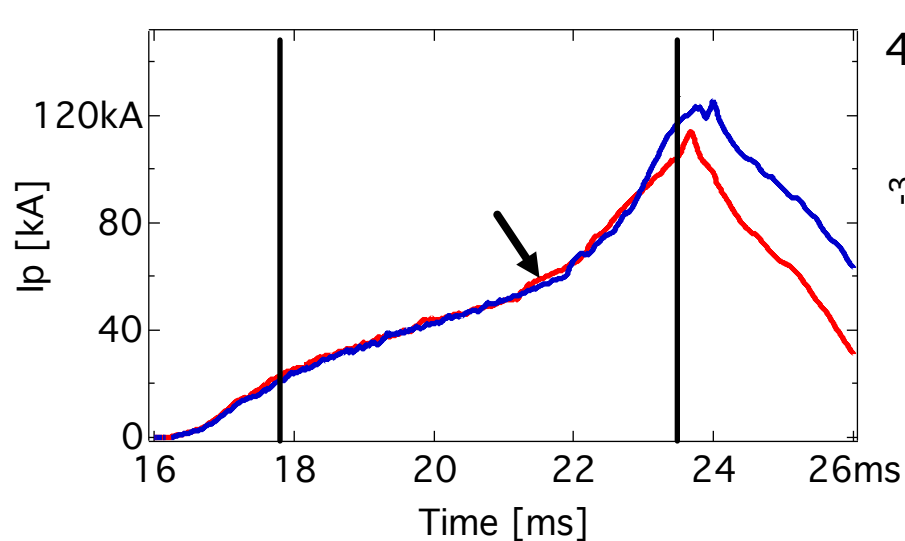
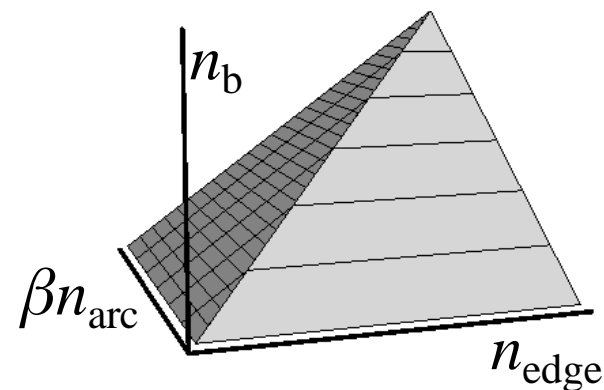
- Increasing  $n_b$  at low  $n_{\text{edge}}$
- Saturation at  $n_b = n_{\text{arc}}/850$





# Interferometer Line-Averaged Density Expected to Trend with $n_{\text{edge}}$

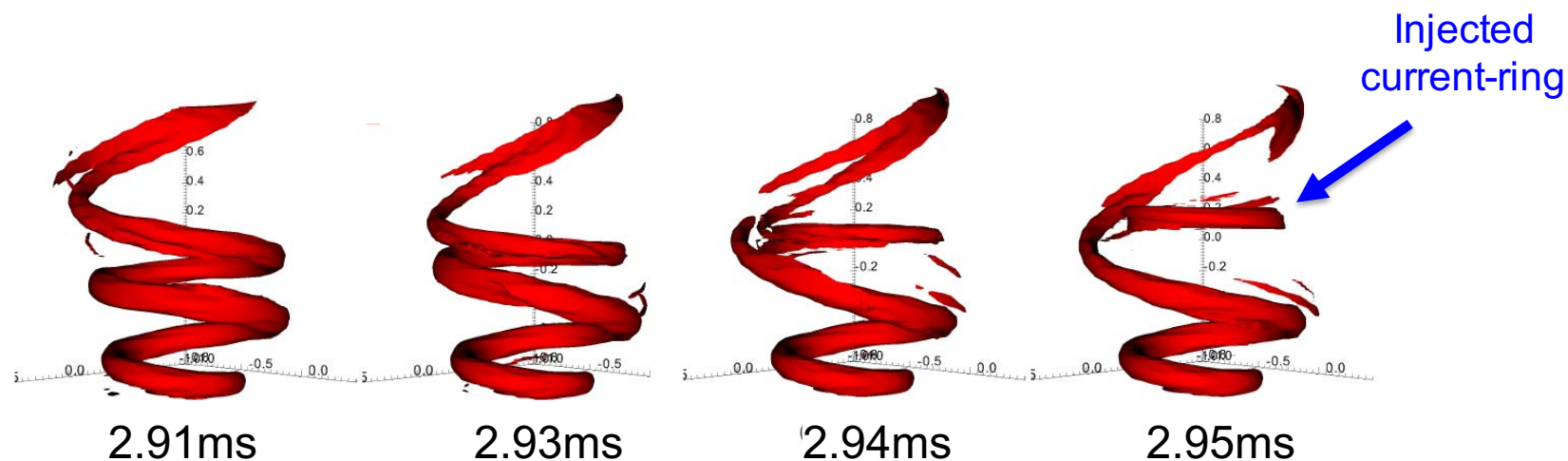
- Interferometer captures linear behavior at low  $n_{\text{edge}}$
- Saturation at  $n_b = n_{\text{arc}}/850$





# NIMROD Simulations Show Reconnecting Streams in Edge

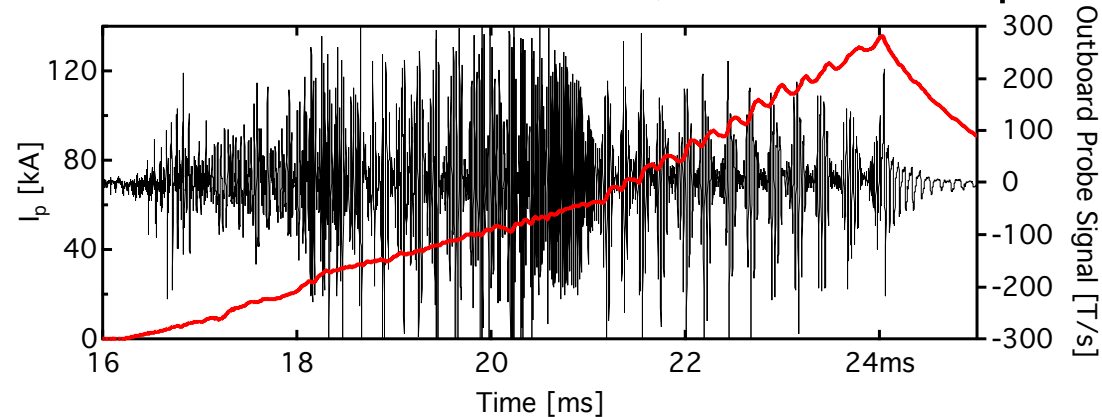
- NIMROD shows\*  $I_p$  growth via intermittent reconnection
  - Coherent current streams exist in edge throughout discharge
  - Adjacent passes reconnect to inject rings into core
- Poloidal flux buildup and  $I_p$  multiplication results



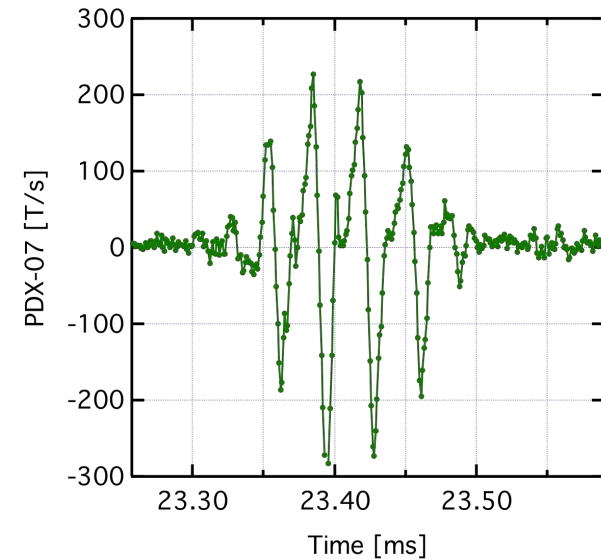
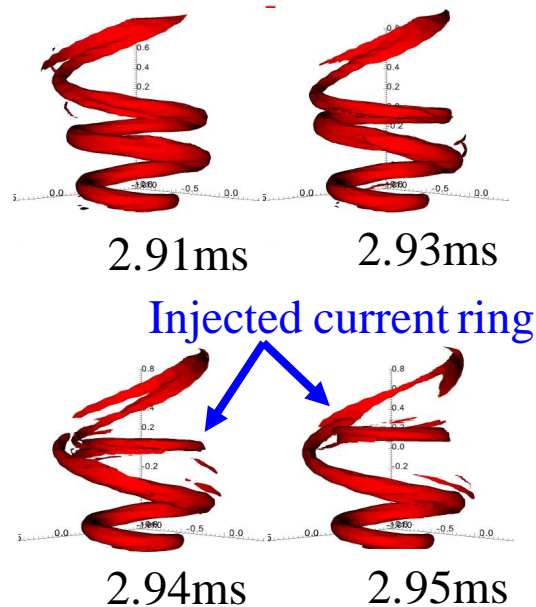


# Connection between Loop Creation, Bursts

- Bursts associated with coherent stream, current buildup



- What could MHD say about existence of beam in edge?







# Hypothesis: dB/dt on PDXs Comes from 'Whirling' (Infinite) Line Source

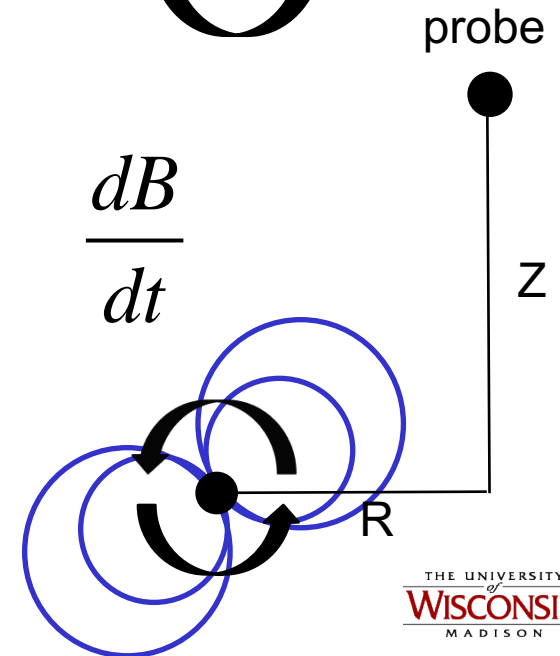
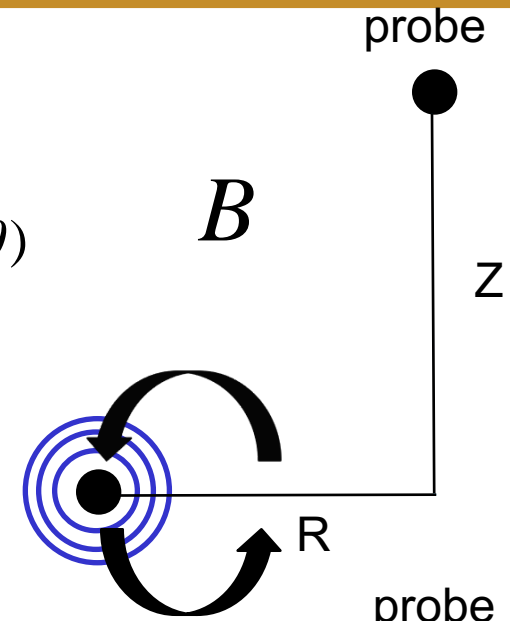
- Assume remote, circular stream rotation

$$\Delta B = \frac{\mu_0 I}{2\pi r_1} - \frac{\mu_0 I}{2\pi r_2} = \frac{\mu_0 I}{2\pi r_1 r_2} (\vec{r}_1 - \vec{r}_2) \approx \frac{\mu_0 I}{2\pi r^2} (r_1 - r_2) \cos(\theta)$$

$$\frac{dB}{dt} \approx \frac{\mu_0 I}{2\pi} \frac{(r_1 - r_2) \cos(\theta)}{dt r^2}$$

$$\frac{dB}{dt} \approx \frac{\mu_0 I}{2\pi} 2\pi r f \frac{\cos(2\pi f t - \arctan(Z/R))}{R^2 + Z^2}$$

- dB/dt structure looks like 'tumbling' dipole
- However, probes measure  $d(B_z)/dt$ , not total dB/dt



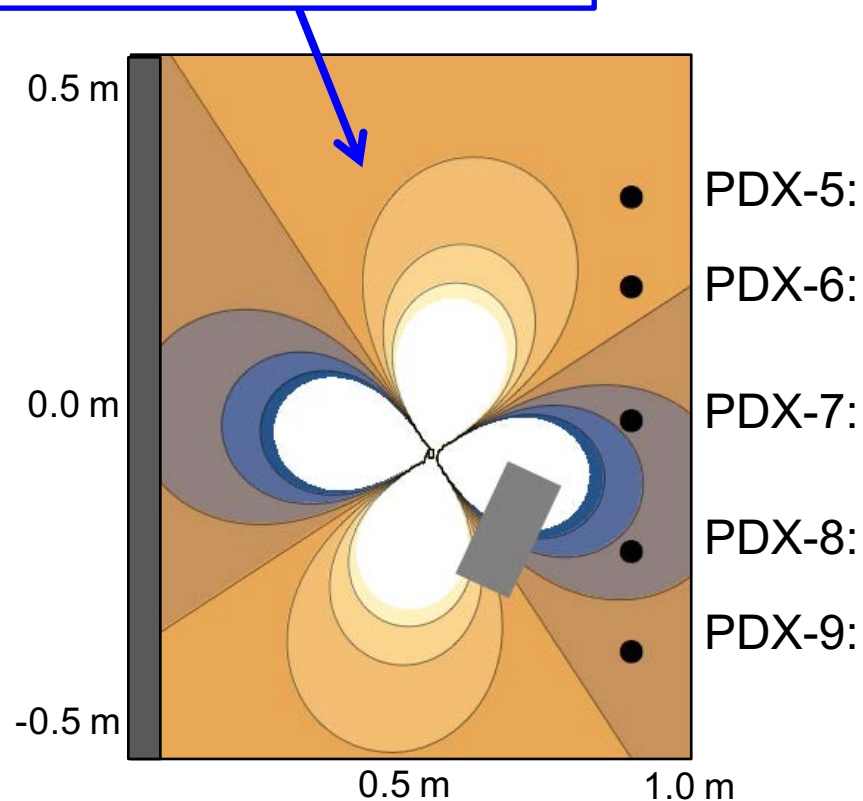


# Hypothesis: dB/dt on PDXs Comes from 'Whirling' (Infinite) Line Source

- Hypothesized  $dB_z/dt$ :

$$\frac{dB_{z,probe}}{dt} = \frac{\mu_0 I r_{motion} f \cos[2\pi ft - 2 \arctan[\frac{z_{probe} - Z_{stream}}{r_{probe} - R_{stream}}]]}{(z_{probe} - Z_{stream})^2 + (r_{probe} - R_{stream})^2}$$

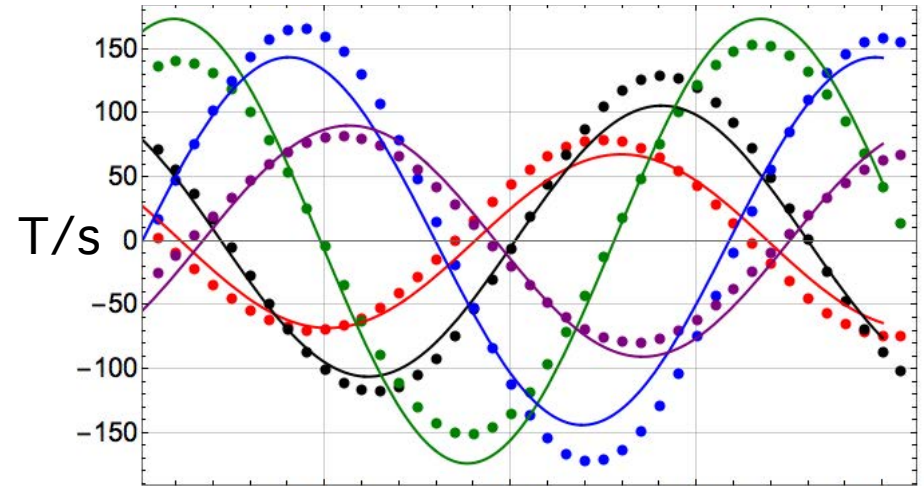
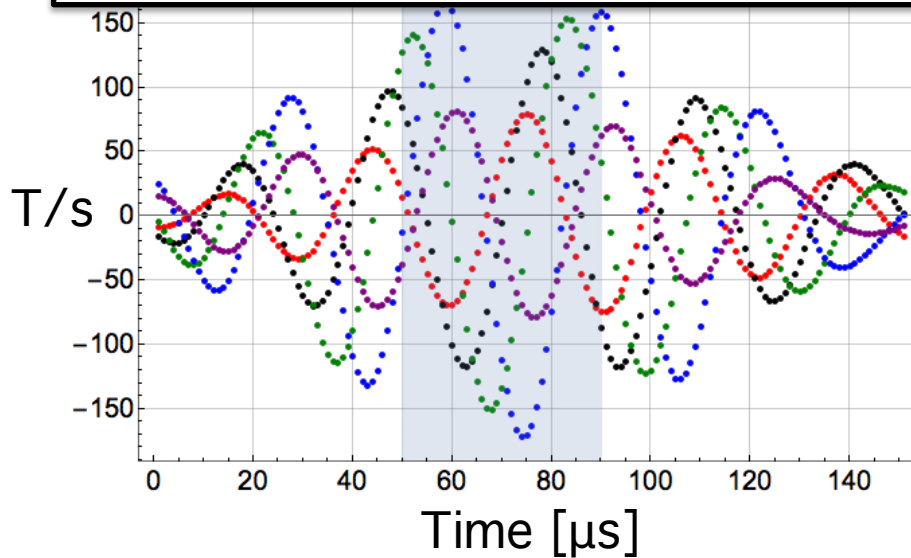
- 4-lobed, fall-off is  $\frac{1}{r^2}$





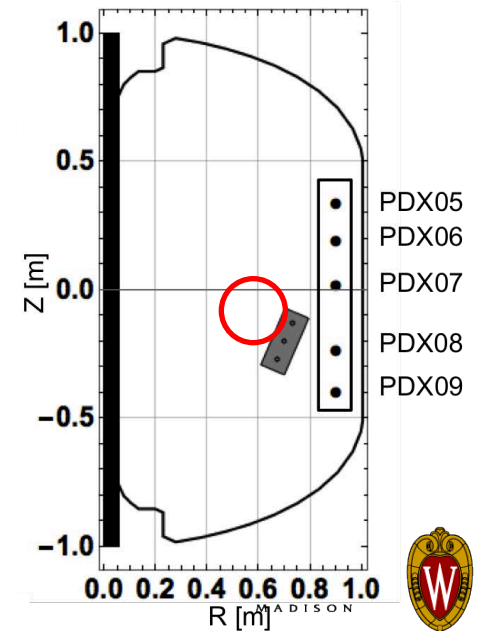
# Fit to Data Looks Promising

PDX05 PDX06 PDX07 PDX08 PDX09



$$\frac{dB_{z,probe}}{dt} = \frac{\mu_0 I r_{motion} f \cos[2\pi ft - 2 \arctan[\frac{z_{probe} - Z_{stream}}{r_{probe} - R_{stream}}]]}{(z_{probe} - Z_{stream})^2 + (r_{probe} - R_{stream})^2}$$

$R_{stream}$	$Z_{stream}$	$r_{motion}$
0.58m	-0.08m	10cm





# Model Inverts for R(t), Z(t)

- Phase difference of 2 signals is

$$\Delta\varphi = 2 \arctan\left[\frac{z_{probe1} - Z_{stream}}{r_{probe1} - R_{stream}}\right] - 2 \arctan\left[\frac{z_{probe2} - Z_{stream}}{r_{probe2} - R_{stream}}\right]$$

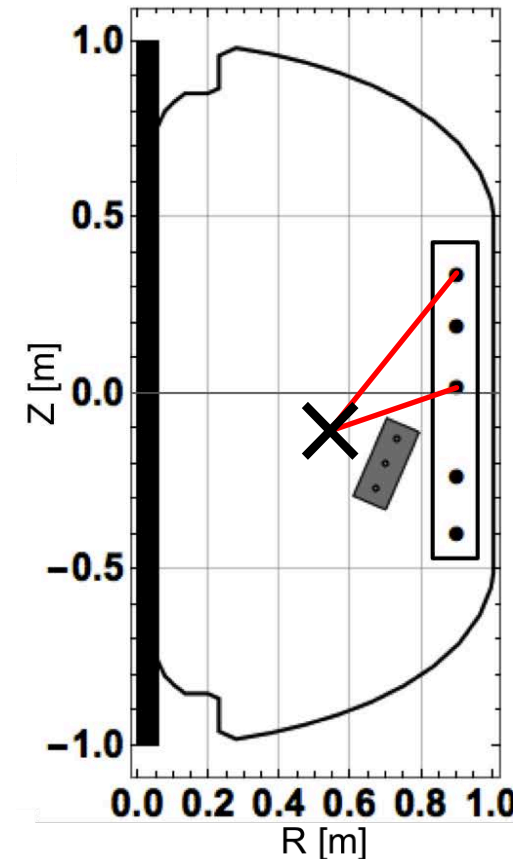
- The ratio of amplitudes of 2 signals

$$T = \frac{\left(z_{probe2} - Z_{stream}\right)^2 + \left(r_{probe2} - R_{stream}\right)^2}{\left(z_{probe1} - Z_{stream}\right)^2 + \left(r_{probe1} - R_{stream}\right)^2}$$

- 2 equations, 2 unknown R, Z of stream:

$$R_{stream} = r_p \pm \frac{\sqrt{T} \Delta z \sin(\Delta\varphi / 2)}{T \pm 2 \cos(\Delta\varphi / 2) \sqrt{T} + 1}$$

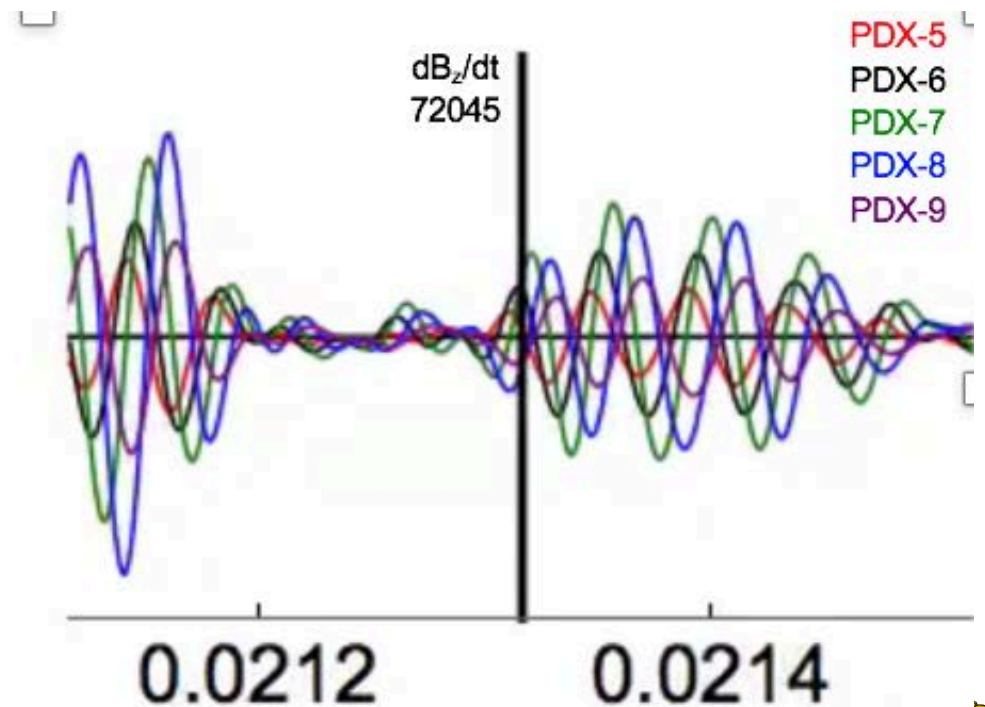
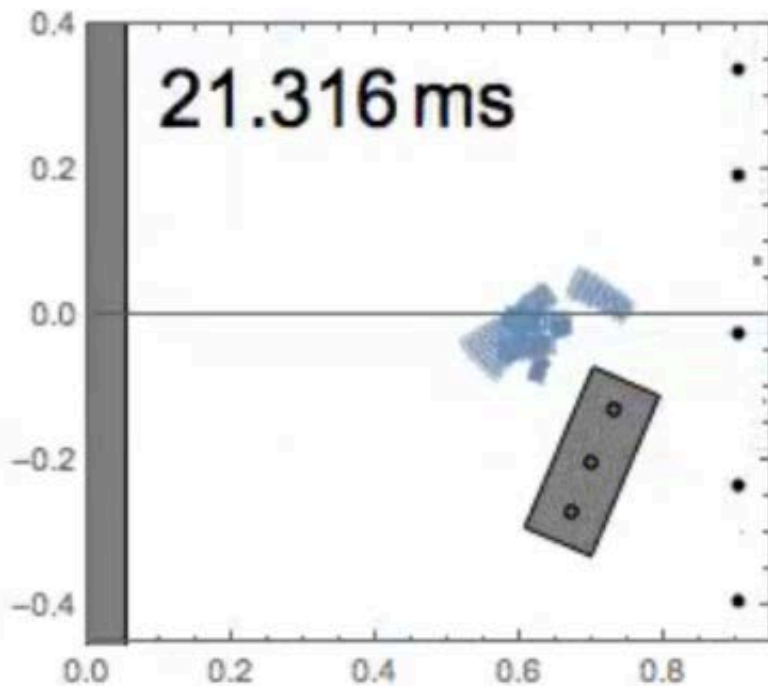
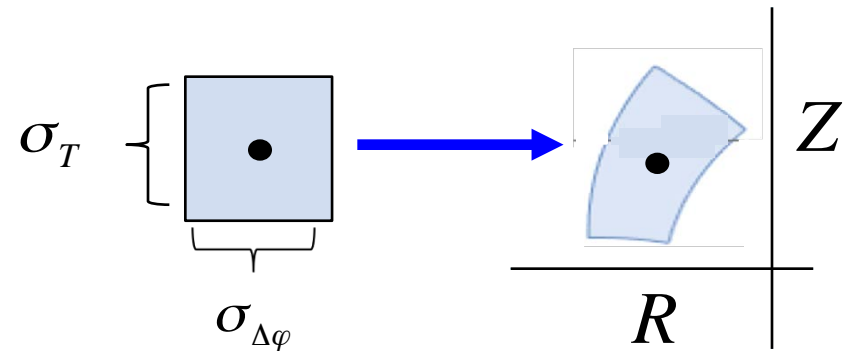
$$Z_{stream} = \frac{z_2 - T \cos(\Delta\varphi) (z_1 + z_2) + z_1 T^2 \pm \sqrt{T} (T - 1) \Delta z \cos(\Delta\varphi / 2)}{T^2 - 2 \cos(\Delta\varphi) T + 1}$$





# $R_{\text{stream}}(t), Z_{\text{stream}}(t)$ is Output

- Combine with uncertainty to yield small R,Z region
- Signal origin appears localized



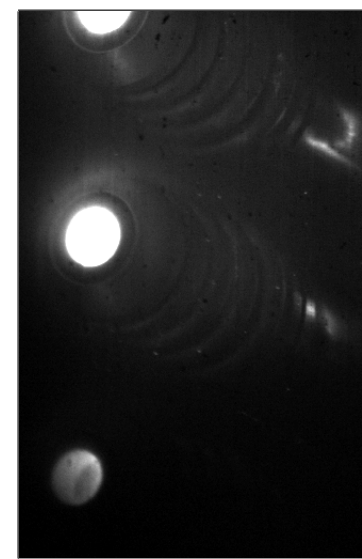


# Managing Plasma-Material Interaction is a Formidable Challenge

- Injector requirements include

- $V_{inj} > 1 \text{ kV}$
- Large  $A_{inj}$ ,  $J_{inj}$
- $\Delta t_{pulse} \sim 10\text{-}100 \text{ ms}$
- Minimal PMI

...all adjacent to tokamak LCFS



- Significant evolution of design to meet physics challenges

- $\sim 3x$  improvement in  $V_{inj}$ ,  
 $\Delta t_{pulse}$

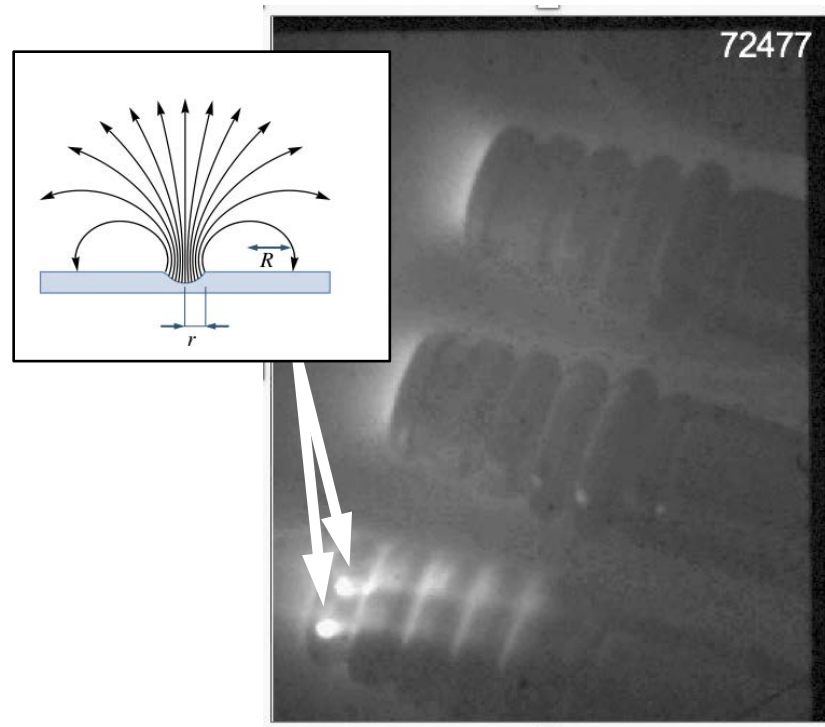
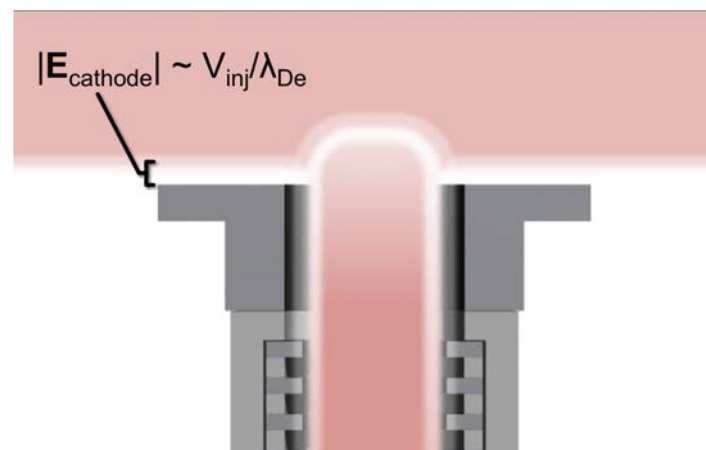






# Spots Move and Interact with Injector Structures

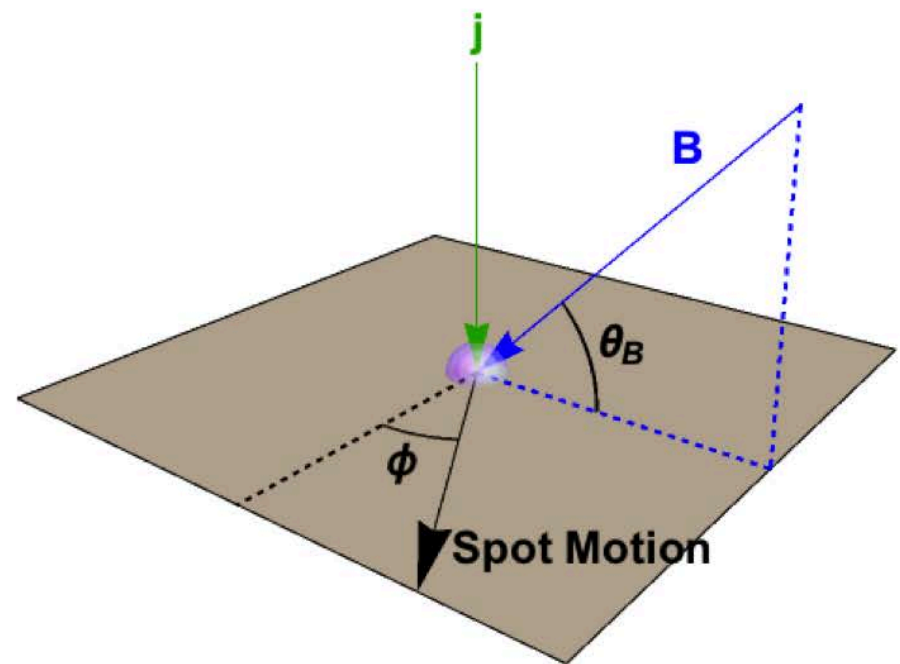
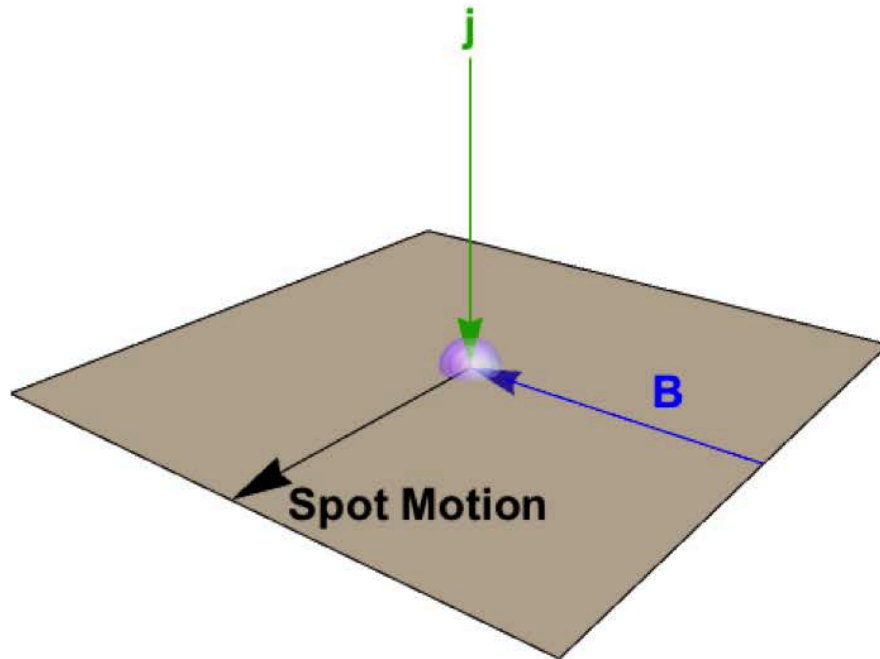
- Breakdown in the presence of plasma occurs  $\sim 10^5$  V/cm
  - At  $10^{18}/\text{m}^3$ , 10eV plasma, this is  $\sim 1\text{kV}$
- Cathode spots, when ignited, roam cathode surface
- Interaction with insulators causes outgassing, damage
- Motion in field can potentially be controlled





# Cathode Spot Motion in a Field

- Spot motion occurs in  $-\mathbf{j} \times \mathbf{B}$  direction, subject to angular displacement,  $\phi$

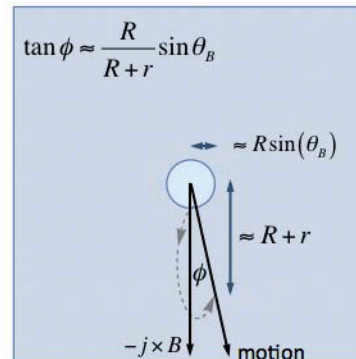
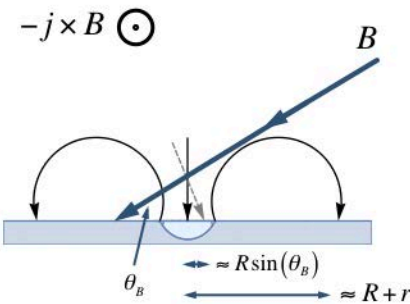
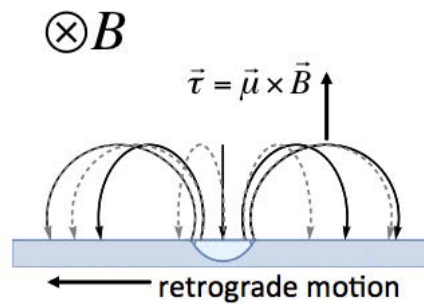
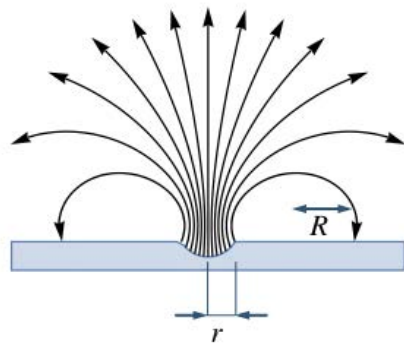






# Barengoltz\* Provides a Model for Motion in Arbitrary B Field

- Based on return currents seen in simulations
- New spots arise due to preferential bombardment by returning electrons
- ‘Small spots’ have  $r \ll R$ , ‘Large spots’ have  $r = R$



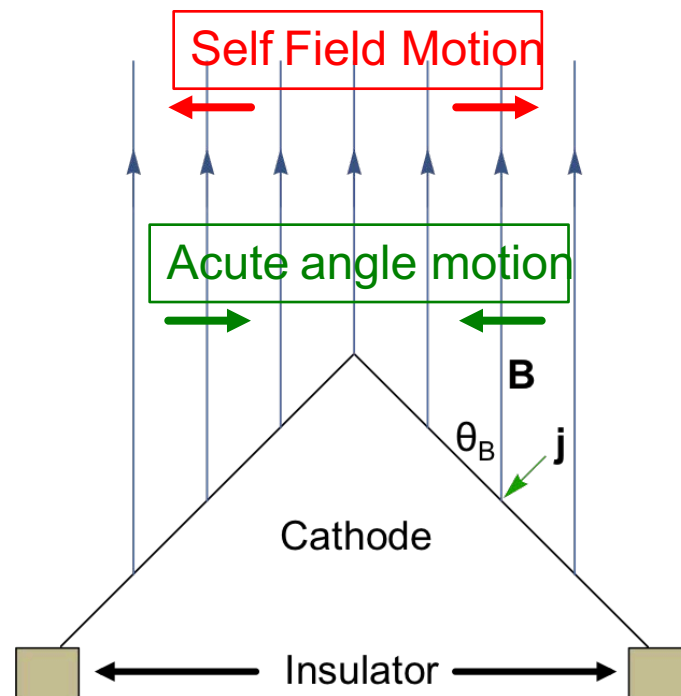
$$\phi \approx \arctan(Q \sin(\theta_B))$$

$$\frac{1}{2} \leq Q \leq 1$$



# Conical Frustum Design Implemented to Mitigate Cathode Spot Damage

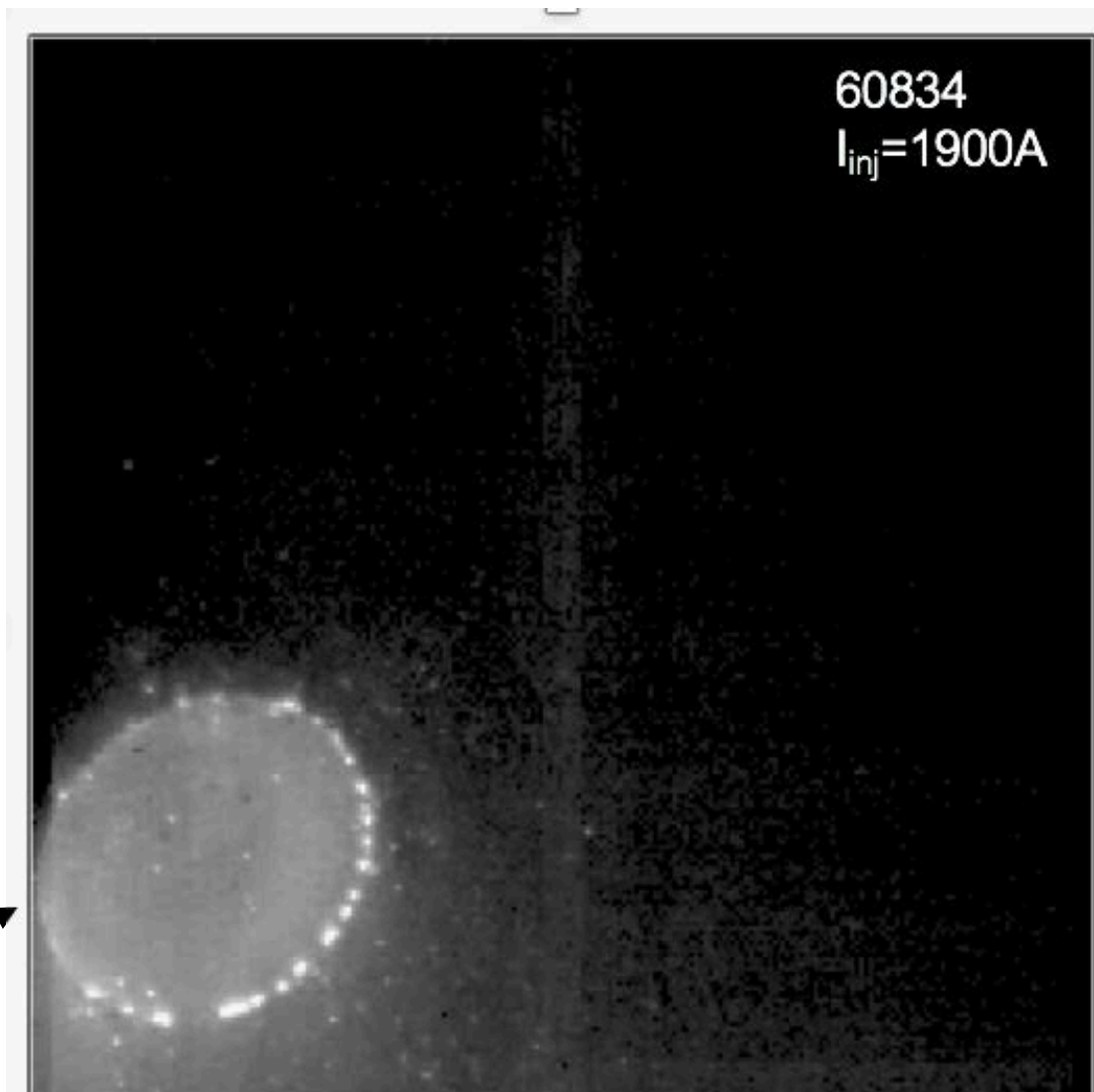
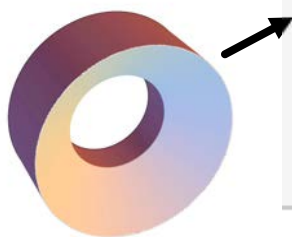
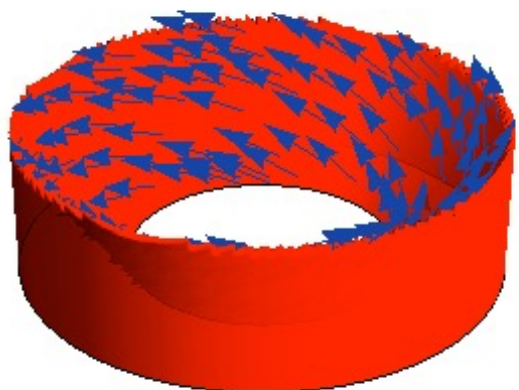
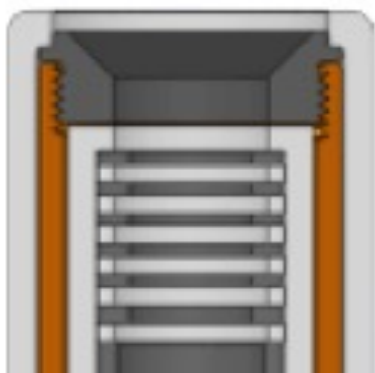
- Conical shape used to induce inward motion of spots
- Self-magnetic field in Pegasus pushes spots outward radially
- Numerical approach necessary to balance these tendencies



- Heuristics are rendered as: 
$$\vec{r}' = q \left( \hat{b} - \hat{n} (\hat{n} \cdot \hat{b}) \right) + \hat{n} \times \hat{b}$$



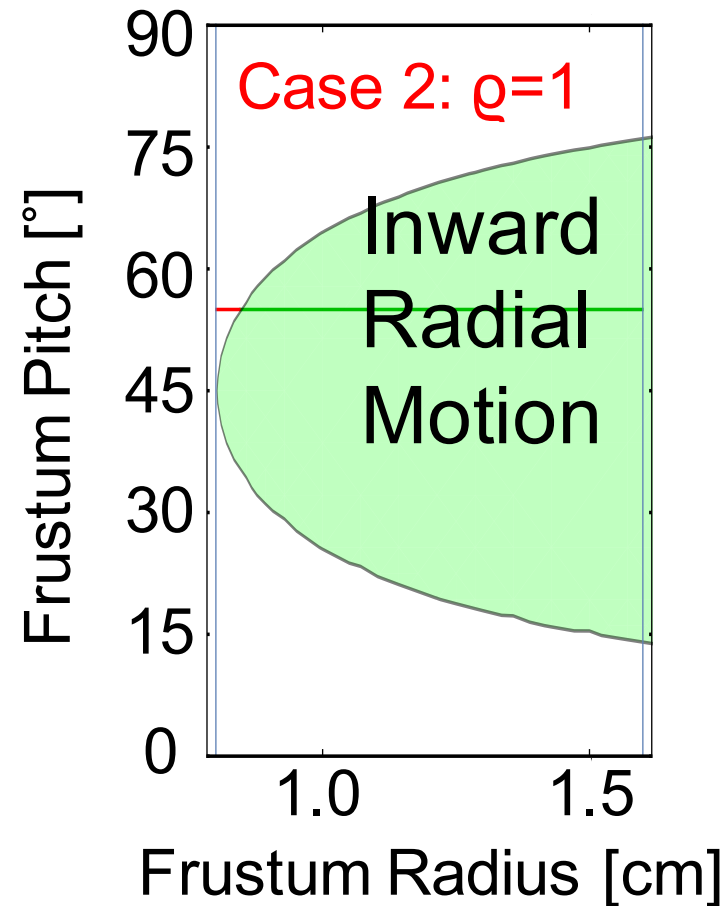
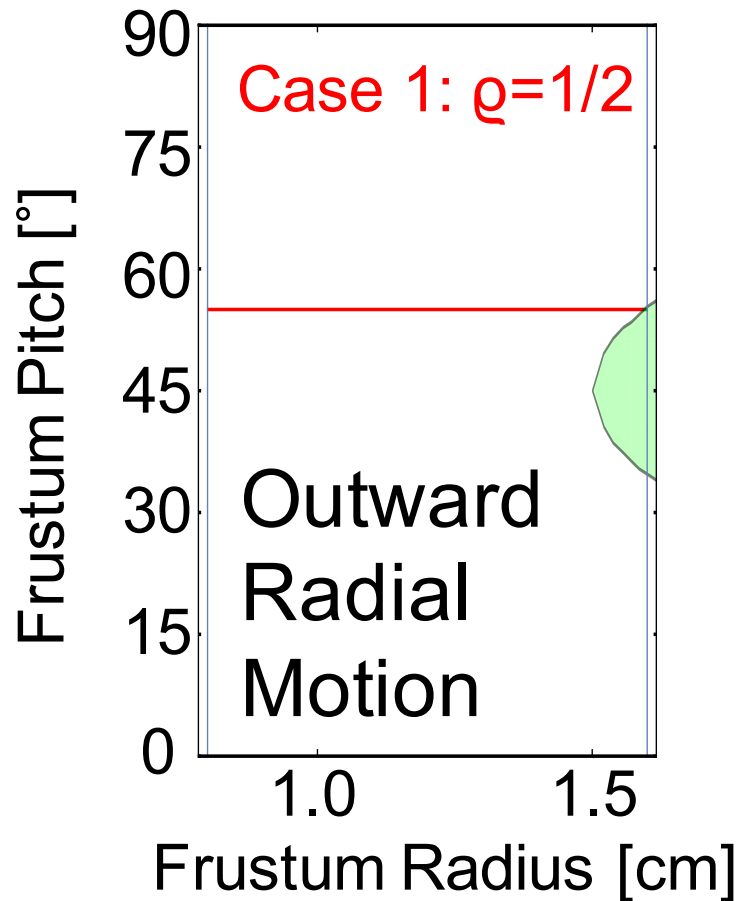
# Motion ALWAYS Outward for Concave Shapes





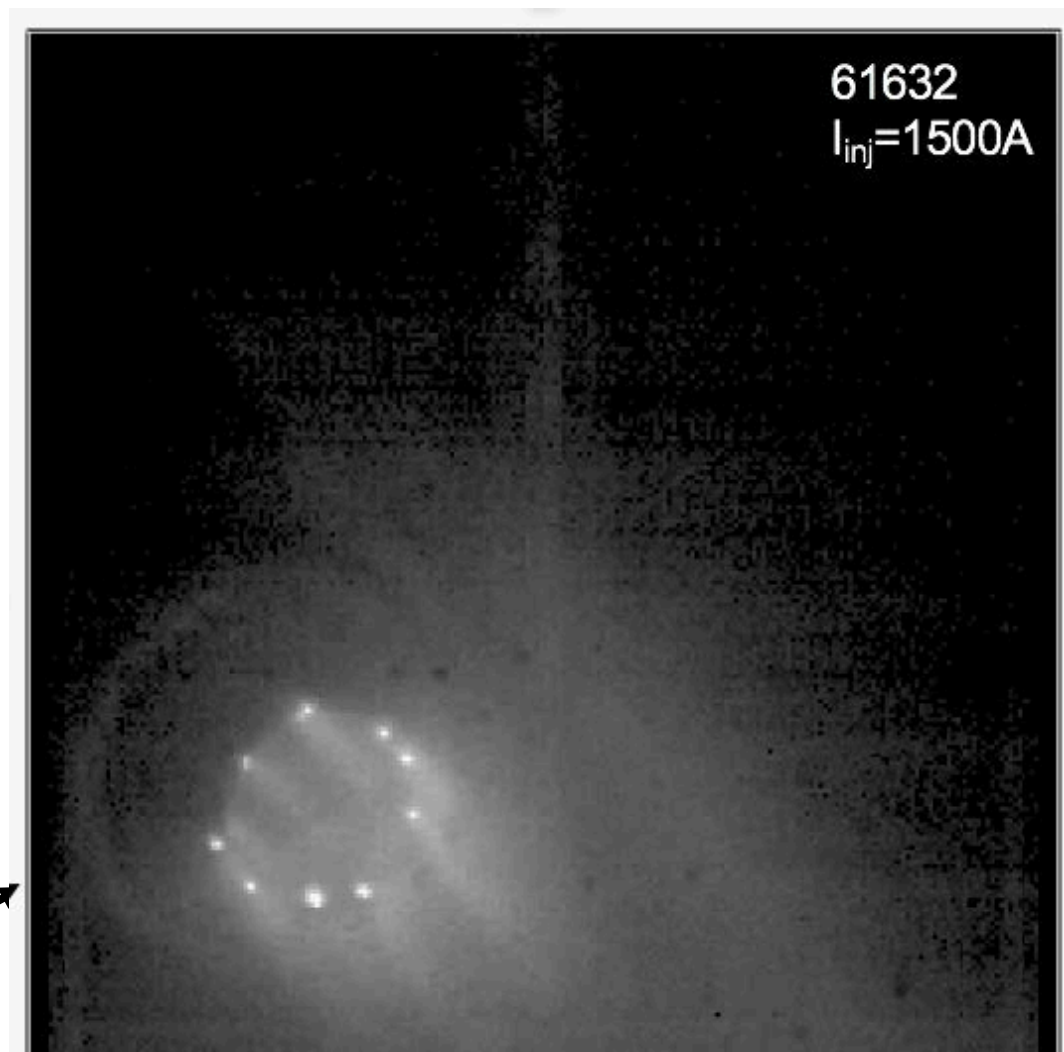
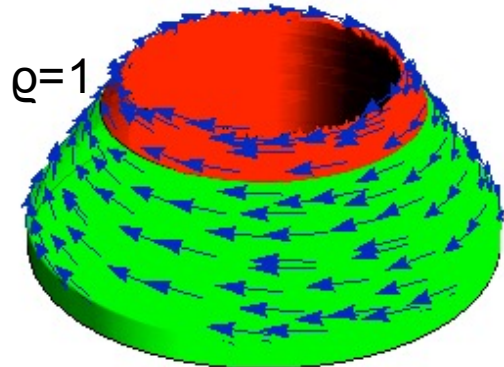
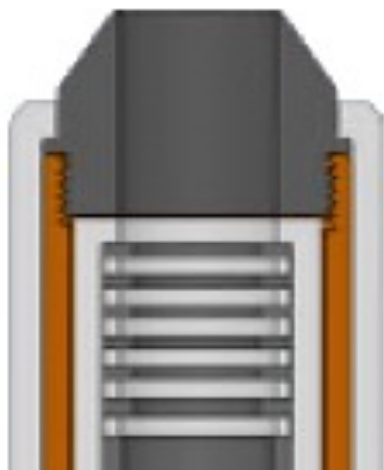
# Convex Shape MIGHT Have Inward Motion

- The sign of  $\vec{r}' = \varrho \left( \hat{b} - \hat{n} (\hat{n} \cdot \hat{b}) \right) + \hat{n} \times \hat{b}$  gives spot radial motion





# Motion is Inward for Convex Cathodes in a Certain Interval





# Summary

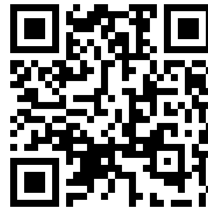
- Limits to max  $I_p$  in LHI depend on  $V_{inj}$ ,  $I_{inj}$ , and are related by

$$I_{inj} = \min[n_{edge}, \beta n_{arc}] e \sqrt{\frac{2e}{m_e}} \sqrt{V_{inj}} A_{inj}$$

- Initial model to understand MHD was created and suggests an oscillating, coherent beam
- Injector design improvements are allowing access to higher power operations

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Reprints:



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