Physics of the Current Injection Process in Localized Helicity Injection

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Pegasus Toroidal Experiment
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Local Helicity Injection (LHI) is a Promising Non-Solenoidal Tokamak Startup Technique

- Unstable current streams form tokamak-like state via Taylor relaxation
- Appears scalable to MA-class startup

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The simple model for helicity injection posits two $I_p$ limits related to the injector circuit:

- **Helicity balance** limit depends on injector voltage $V_{\text{inj}}$:

  $$I_p \leq \frac{A_p}{2\pi R_0 \langle \eta \rangle} \left( V_{\text{ind}} + \frac{A_{\text{INJ}} B_{\phi,\text{inj}}}{\Psi_T} V_{\text{inj}} \right)$$

- **Taylor relaxation** limit depends on current $I_{\text{inj}}$:

  $$I_p \leq f(\varepsilon, \delta, \kappa) \sqrt{\frac{\kappa A_p I_{\text{TTR}} I_{\text{inj}}}{2\pi R_0 w}}$$

Injector impedance describes the relationship between $I_{\text{inj}}$ and $V_{\text{inj}}$:

- Determined by plasma physics
- Determines feasibility and power requirements for scaling up LHI
Edge Current Injection is Straightforward Conceptually

- Injection in Pegasus accomplished with arc plasma guns

- 2-stage circuit: bias and arc circuits “daisy chained”

- Voltage (~1 kV) is determined by plasma physics
Double Layer Sheath: A Promising Framework for Understanding Injector Impedance

- Two space-charged layers “sandwiched” to each other

- Width of space charge set by plasma, order $\lambda_{De}$

$$J = \frac{4}{9} 1.865 \varepsilon_0 \left(1 + \sqrt{\frac{m_e}{m_i}}\right) \left(\frac{2e}{m_e}\right)^{\frac{1}{2}} \frac{V^2}{\ell_{DL}^2}$$

$$\ell_{DL}^2 = (\lambda_{De} \cdot \chi)^2 \Rightarrow I \sim n_{DL} V^\frac{3}{2}$$
I-V Characteristics Show 2 Regimes

- Typical I-V relationship shows two power law regimes, $I_{\text{inj}} \sim V_{\text{inj}}^{3/2}, V_{\text{inj}}^{1/2}$
Injector Impedance Has Fueling Dependence

- Deuterium gas flow rate into the source plasma scanned
- $I_{\text{inj}}/V^{1/2}$ increases with gas flow

![Graph showing $I_{\text{inj}}$ vs. $V_{\text{inj}}$ with $D_2$ flow rate as a parameter.]

$V_{\text{inj}}$ [V] vs. $I_{\text{inj}}$ [A]

$D_2$ Flow Rate [Torr-L/s]
- 2700
- 2300
- 2000
- 1900
- 1600
- 1400
- 720

$I_{\text{inj}}/V^{1/2}$ vs. $D_2$ Flow Rate [Torr-L/s]

$V_{\text{inj}}$ [V] vs. $I_{\text{inj}}$ [A]

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1: Expanding Double Layer Yields $I \sim n_{DL} V^{1/2}$

- Simulations* find when $V_{DL}/T_e >> 1$:

$$\ell_{DL} \sim \lambda_{De} \sqrt{V_{DL}/T_e}$$

Expected I-V relation is:

$$I_{inj} \sim n_{arc} \sqrt{V_{inj}}$$

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2: Beam Neutralization Yields $I_{inj} \sim n_{edge} V_{inj}^{1/2}$

- Electron beam propagation requires drift space ions neutralize electrons: $n_b \leq n_i$

- Typical beam values imply beam density $n_b \sim 10^{18} m^{-3}$ - comparable to edge density $n_{edge}$!

- Assume drift space has same density as edge: $n_i \approx n_{edge}$

\[
I_{inj} = n_b e v_e A_{inj} \leq n_i e \sqrt{\frac{2eV_{inj}}{m_e} A_{inj}} \sim n_{edge} \sqrt{V_{inj}}
\]

\[
I_{inj} \sim n_{edge} \sqrt{V_{inj}}
\]
Minimum of both limits is applicable:

Sheath expansion: \[ I_{inj} \sim n_{arc} \sqrt{V_{inj}} \]

Quasineutrality: \[ I_{inj} \sim n_{edge} \sqrt{V_{inj}} \]

Impedance Model:
\[ I_{inj} = \text{Min}[n_{edge}, \beta n_{arc}] e^{\frac{2eV_{inj}}{m_e}} A_{inj} \]
Ohmic Plasmas Created to Test Model via Measured $n_{\text{edge}}$, $n_{\text{arc}}$

$n_{\text{edge}}$:
- Measured with Langmuir probe behind injector limiter
- Controlled with edge fueling

$n_{\text{arc}}$:
- Measured via Stark broadening of H-$\delta$ in arc channel
- Controlled with injector fueling

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• Arc density $n_{\text{arc}}$ scanned
• Entire scan consistent with sheath expansion: $n_b \sim \beta n_{\text{arc}}$
  where $\beta = 1/850$

$$\beta n_{\text{arc}}$$

$$n_{\text{edge}}$$

$\triangle n_{\text{edge}}$

$\bullet n_b$

$n_b = n_{\text{arc}}/850$

Density $[\text{m}^{-3}]$

$12 \times 10^{18}$

$10^{15}$

$10^{12}$

$10^{9}$

$10^{6}$

$10^{3}$

$10^{0}$

$0.1$ $2$ $4$ $6$ $8$ $10 \times 10^{21}$

$n_{\text{arc}} [\text{m}^{-3}]$
• Increasing $n_b$ at low $n_{\text{edge}}$

• Saturation at $n_b = n_{\text{arc}} / 850$
Interferometer Line-Averaged Density Expected to Trend with $n_{\text{edge}}$

- Interferometer captures linear behavior at low $n_{\text{edge}}$
- Saturation at $n_b = n_{\text{arc}}/850$

\[ n_b \approx 3 \times 10^{21} \text{ m}^{-3} \]

\[ n_{\text{arc}} \approx 3 \times 10^{21} \text{ m}^{-3} \]
NIMROD Simulations Show Reconnecting Streams in Edge

- NIMROD shows $I_p$ growth via intermittent reconnection
  - Coherent current streams exist in edge throughout discharge
  - Adjacent passes reconnect to inject rings into core

- Poloidal flux buildup and $I_p$ multiplication results


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Connection between Loop Creation, Bursts

- Bursts associated with coherent stream, current buildup

- What could MHD say about existence of beam in edge?
Hypothesis: dB/dt on PDXs Comes from ‘Whirling’ (Infinite) Line Source

- Assume remote, circular stream rotation

\[ \Delta B = \frac{\mu_0 I}{2\pi r_1} - \frac{\mu_0 I}{2\pi r_2} = \frac{\mu_0 I}{2\pi r_1 r_2} (\vec{r}_1 - \vec{r}_2) \approx \frac{\mu_0 I}{2\pi r^2} (r_1 - r_2) \cos(\theta) \]

\[ \frac{dB}{dt} \approx \frac{\mu_0 I}{2\pi} \left( \frac{r_1 - r_2}{r^2} \right) \cos(\theta) \]

\[ \frac{dB}{dt} \approx \frac{\mu_0 I}{2\pi} 2\pi f \cos(2\pi ft - \arctan(Z / R)) \]

\[ \frac{dB}{dt} \approx \frac{\mu_0 I}{2\pi} \frac{2\pi ft - \arctan(Z / R)}{R^2 + Z^2} \]

- dB/dt structure looks like ‘tumbling’ dipole

- However, probes measure d(B_z)/dt, not total dB/dt

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Hypothesis: dB/dt on PDXs Comes from ‘Whirling’ (Infinite) Line Source

- Hypothesized dB_z/dt:

\[
\frac{dB_{z,\text{probe}}}{dt} = \frac{\mu_0 I r_{\text{motion}} f \cos[2\pi ft - 2\arctan\left(\frac{z_{\text{probe}} - Z_{\text{stream}}}{r_{\text{probe}} - R_{\text{stream}}}\right)]}{\left(\frac{z_{\text{probe}} - Z_{\text{stream}}}{r_{\text{probe}} - R_{\text{stream}}}\right)^2 + \left(\frac{z_{\text{probe}} - Z_{\text{stream}}}{r_{\text{probe}} - R_{\text{stream}}}\right)^2}
\]

- 4-lobed, fall-off is \(\frac{1}{r^2}\)
Fit to Data Looks Promising

\[ \frac{dB_{z,\text{probe}}}{dt} = \mu_0 I_{\text{motion}} f \cos(2\pi ft - 2 \arctan\left[ \frac{z_{\text{probe}} - Z_{\text{stream}}}{r_{\text{probe}} - R_{\text{stream}}} \right]) \]

\[ \left( z_{\text{probe}} - Z_{\text{stream}} \right)^2 + \left( r_{\text{probe}} - R_{\text{stream}} \right)^2 \]

<table>
<thead>
<tr>
<th>( R_{\text{stream}} )</th>
<th>( Z_{\text{stream}} )</th>
<th>( r_{\text{motion}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.58m</td>
<td>-0.08m</td>
<td>10cm</td>
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Model Inverts for $R(t)$, $Z(t)$

- Phase difference of 2 signals is
  \[
  \Delta \varphi = 2 \arctan \left( \frac{z_{\text{probe}1} - Z_{\text{stream}}}{r_{\text{probe}1} - R_{\text{stream}}} \right) - 2 \arctan \left( \frac{z_{\text{probe}2} - Z_{\text{stream}}}{r_{\text{probe}2} - R_{\text{stream}}} \right)
  \]

- The ratio of amplitudes of 2 signals
  \[
  T = \frac{\left( z_{\text{probe}2} - Z_{\text{stream}} \right)^2 + \left( r_{\text{probe}2} - R_{\text{stream}} \right)^2}{\left( z_{\text{probe}1} - Z_{\text{stream}} \right)^2 + \left( r_{\text{probe}1} - R_{\text{stream}} \right)^2}
  \]

- 2 equations, 2 unknown $R$, $Z$ of stream:
  \[
  R_{\text{stream}} = r_p \pm \frac{\sqrt{T \Delta z \sin(\Delta \varphi / 2)}}{T \pm 2 \cos(\Delta \varphi / 2) \sqrt{T} + 1}
  \]
  \[
  Z_{\text{stream}} = \frac{z_2 - T \cos(\Delta \varphi)(z_1 + z_2) + z_1 T^2 \pm \sqrt{T \left( T - 1 \right) \Delta z \cos(\Delta \varphi / 2)}}{T^2 - 2 \cos(\Delta \varphi) T + 1}
  \]
\( R_{\text{stream}}(t), Z_{\text{stream}}(t) \) is Output

- Combine with uncertainty to yield small R,Z region
- Signal origin appears localized

\[ \sigma_T \]

\[ \sigma_{\Delta \phi} \]

\[ R \]

\[ Z \]
Managing Plasma-Material Interaction is a Formidable Challenge

- Injector requirements include
  - $V_{\text{inj}} > 1 \text{ kV}$
  - Large $A_{\text{inj}}, J_{\text{inj}}$
  - $\Delta t_{\text{pulse}} \sim 10\text{-}100 \text{ ms}$
  - Minimal PMI
  - ...all adjacent to tokamak LCFS

- Significant evolution of design to meet physics challenges
  - $\sim 3x$ improvement in $V_{\text{inj}}, \Delta t_{\text{pulse}}$

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Spots Move and Interact with Injector Structures

- Breakdown in the presence of plasma occurs $\sim 10^5$ V/cm
  - At $10^{18}/m^3$, 10eV plasma, this is $\sim 1kV$

- Cathode spots, when ignited, roam cathode surface

- Interaction with insulators cases outgassing, damage

- Motion in field can potentially be controlled
Cathode Spot Motion in a Field

- Spot motion occurs in $-j \times B$ direction, subject to angular displacement, $\phi$
Barengoltz* Provides a Model for Motion in Arbitrary B Field

- Based on return currents seen in simulations
- New spots arise due to preferential bombardment by returning electrons
- ‘Small spots’ have \( r << R \), ‘Large spots’ have \( r = R \)

\[
\phi \approx \arctan \left( Q \sin(\theta_B) \right)
\]

\[
\frac{1}{2} \leq Q \leq 1
\]

*Zh. Tekh. Fiz. 68, 60–64 (June 1998)
Conical Frustum Design Implemented to Mitigate Cathode Spot Damage

- Conical shape used to induce inward motion of spots
- Self-magnetic field in Pegasus pushes spots outward radially
- Numerical approach necessary to balance these tendencies

Heuristics are rendered as:

$$\vec{r}' = q\left(\hat{b} - \hat{n}(\hat{n} \cdot \hat{b})\right) + \hat{n} \times \hat{b}$$

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Motion ALWAYS Outward for Concave Shapes

60834
I_{inj}=1900A

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Convex Shape MIGHT Have Inward Motion

- The sign of \( \ddot{r}' = \varrho \left( \hat{b} - \hat{n} (\hat{n} \cdot \hat{b}) \right) + \hat{n} \times \hat{b} \) gives spot radial motion.

**Case 1:** \( \varrho = \frac{1}{2} 

**Case 2:** \( \varrho = 1 

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Motion is Inward for Convex Cathodes in a Certain Interval

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• Limits to max $I_p$ in LHI depend on $V_{INJ}$, $I_{INJ}$, and are related by

$$I_{inj} = \min[n_{edge}, \beta n_{arc}] e \sqrt{\frac{2e}{m_e}} \sqrt{V_{inj}} A_{inj}$$

• Initial model to understand MHD was created and suggests an oscillating, coherent beam

• Injector design improvements are allowing access to higher power operations